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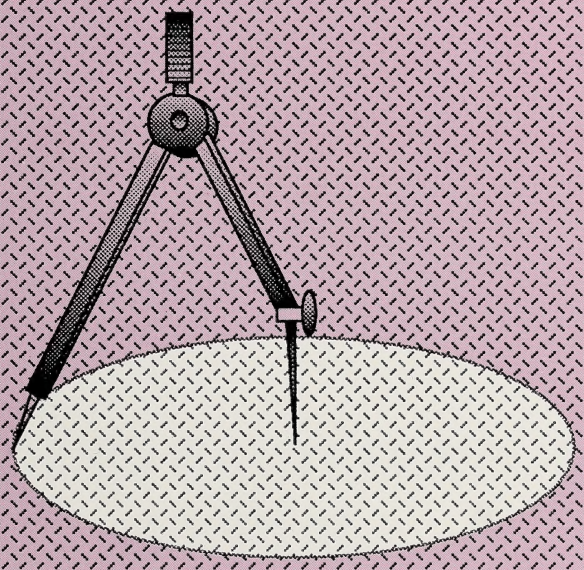
MATHEMATICS 23

LEARNING FACILITATOR'S MANUAL

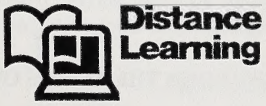


Distance Learning

Alberta EDUCATION

Mathematics 23

LEARNING FACILITATOR'S MANUAL



Alberta
EDUCATION

NOTE: This Mathematics 23 Learning Facilitator's Manual contains answers to teacher-assessed assignments and the Final Test; therefore, it should be kept secure by the teacher. Until these assignments and the Final Test are assigned in a supervised situation, they should not be accessible to students. The answers should be stored securely by the teacher at all times.

Mathematics 23
Learning Facilitator's Manual
Modules 1–8
Alberta Distance Learning Centre
ISBN 0-7741-0010-9

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Final Test

Questionnaire

Introduction

A survey of these course materials will confirm that this new learning package has been specially designed for many kinds of teachers working in a variety of situations.

Which Category Do You Fit?

- ❒ Small Schools Teacher
  - ❒ inexperienced
  - ❒ experienced, but in other subject areas
  - ❒ experienced in mathematics but wanting to try a different approach
- ❒ Distance Learning Teacher
  - ❒ travelling to schools within the jurisdiction
  - ❒ using facsimile and teleconferencing to teach students within the area
- ❒ Larger Schools Teacher
  - ❒ inexperienced
  - ❒ experienced in mathematics but wanting to try a different approach

Because these materials have been created by experienced classroom teachers and distance learning specialists, they have many advantages for students and teachers regardless of their situation.

Advantages for Students

- incorporates a strong learner-centered philosophy
- promotes such qualities in the learner as autonomy, independence, flexibility
- is developed through media which suits the needs and circumstances of the learner
- reflects the experiential background of Alberta students
- opens up opportunities by overcoming barriers that result from geographical location
- promotes individualized learning, allowing learners to work at their own pace

Advantages for Teachers

- allows teachers maximum teaching time and minimizes preparation time
- includes different routes through the materials to suit different learners
- incorporates a wide range of teaching strategies, in particular those using independent and individual learning
- delivers curriculum designed by education specialists that reflects Alberta Education Program of Studies with an emphasis on Canadian content
- provides learning materials which are upwardly compatible with advanced educational technology

Does it sound like something you could use?

The student materials are not the only components designed for independent, guided instruction; so is this Learning Facilitator's Manual. It begins with an overview of the current Alberta Education Program of Studies for Mathematics. This summary is included for inexperienced teachers or those teachers who have found themselves teaching mathematics when their training is in other subject areas. This brief summary is not meant to replace the Alberta Education Program of Studies, but rather to help teachers confirm the highlights of the program.

Other parts of this introduction have also been included to help teachers become familiar with this new courseware and how they might want to use it in their classroom.

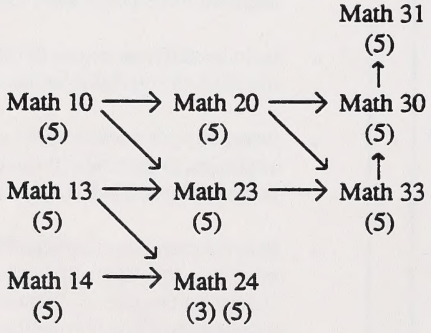
Beyond the introduction, the guide itself contains answers, models, explanations and other tips generated by the teachers who authored this course.

The courseware and LFM are the products of experienced classroom teachers and distance learning specialists. It is the hope of these teachers that their experience can be shared with those who want to take advantage of it.

Overview of Program of Studies

A knowledge of mathematics is essential in many other areas of learning. With available information and the use of technology the emphasis will not be so much on memorizing formulae but on how to use mathematical information to solve problems. You, the learner, will be expected to construct, investigate, translate, discuss, analyze and formulate mathematics in a dynamic way.

The Mathematics 10 - 20 - 30 - 31 program is the academic series leading to university entrance. The Mathematics 13 - 23 - 33 program is the vocational series leading to technical institute entrance. The Mathematics 14 - 24 program is a general series suitable for students who have experienced difficulties in previous mathematics courses. The complete high school mathematics program is illustrated below along with recommended transfer points. The number in brackets indicates credits.



Your mathematics course is divided into units. Each unit covers a major area of mathematical study, and is divided further into specific content areas called topics. Each unit has its own Assignment Booklet which you complete and submit at the end of that unit.

Some of our distance learning courses are divided into sections within modules. If you receive letters or other materials from Alberta Correspondence School which refer to modules, please remember that a module is the same as a unit and a section is the same as a topic.

Module	=	Unit

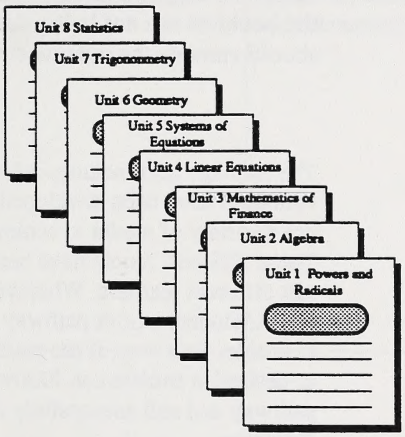
Section	=	Topic

Structure of the Learning Package

Basic Design

This new learning package involves many other components in addition to the Learning Facilitator's Manual.

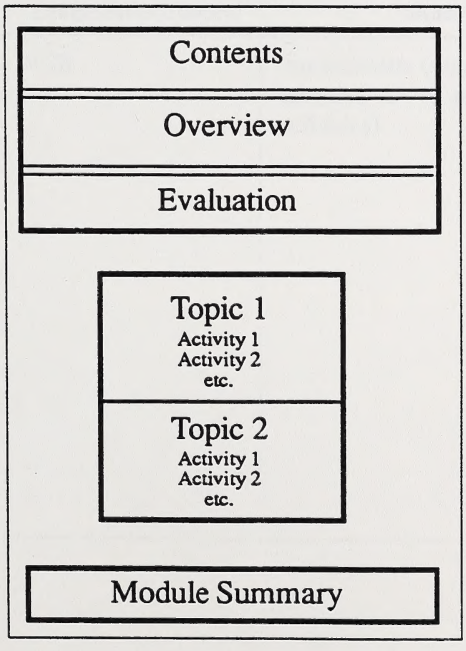
Units



The print components involve many booklets called units. These units contain guided activities that instruct students in a relevant realistic setting.

The units have been specially designed to promote such qualities in the learner as autonomy, independence and flexibility. Writers have incorporated such teaching strategies as working from the concrete to the abstract, linking the old to the new, getting students actively involved, using advance, intermediate and post organizers and many other techniques to enable learners to learn on their own for at least some of the time.

The structure of the Unit Booklets follows a systematic design. Each unit begins with a detailed table of contents which shows the students all the main steps. It acts as an organizer for students. The overview introduces the unit title or theme. A graphic representation has been included to help visual learners and poor readers. The introduction also states the weightings of each assignment.

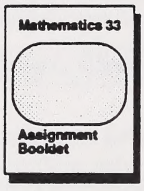


The body of the unit is made up of two or more closely related topics. Each topic contains student activities that develop skills and knowledge centered around a theme.

The activities may involve print, audio and video formats. At times, students and learning facilitators are allowed to choose the activity that best suits the student's needs and interests. Other activities such as the Extra Help and Extensions are optional pathways. This flexibility caters to each student's personal situation.

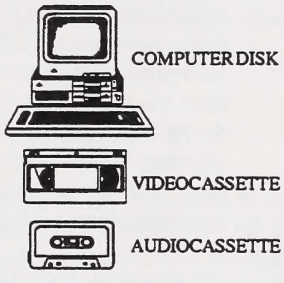
The summary focuses on the skills and strategies that the student has learned.

Assignment Booklet



Accompanying each unit is an Assignment Booklet. The activities in these booklets can be used for formative and for summative assessments. The students should complete the Assignment Booklet when they have thoroughly reviewed the unit materials. The Assignment Booklet has been designed for classroom use, for "faxing," or for mailing. **If the booklets are not being mailed, you should remove the outside cover.**

Media



The package also includes reference to media. Pathways have been developed so students can use a variety of media to achieve the objective. These different routes have been included to suit different learners. Wherever videos have been included, a print pathway is also available. This way, if the media isn't available or desired, a student can follow the print pathway and still successfully achieve the objective.

Materials, Media and Equipment

Mandatory Components

Equipment (Hardware)	Media	Materials
audiocassette player	prepared audiocassette (comes with learning package)	**Unit Booklets (8)** **Assignment Booklets (8)** **Geometry Set** **Scientific calculator**

Optional Components

Equipment (Hardware)	Media	Materials
VCR	videocassette (suggestions included as well as related activities)	

Using This Learning Package in the Classroom

Conventional Classroom

Whether your classroom has desks in rows or tables in small groups, you may be most comfortable with a learning system that you can use with all your students in a paced style. In other words, you may want a package that will suit all of your students so they can move through the materials as one group or several small groups. Because these materials contain different routes or pathways within each unit, they can address various learning styles and preferences. The materials also include many choices within the activities to cater to different thinking levels and ability levels. Because of their versatility and flexibility, these materials can easily suit a conventional classroom.

Open-Learning Classroom

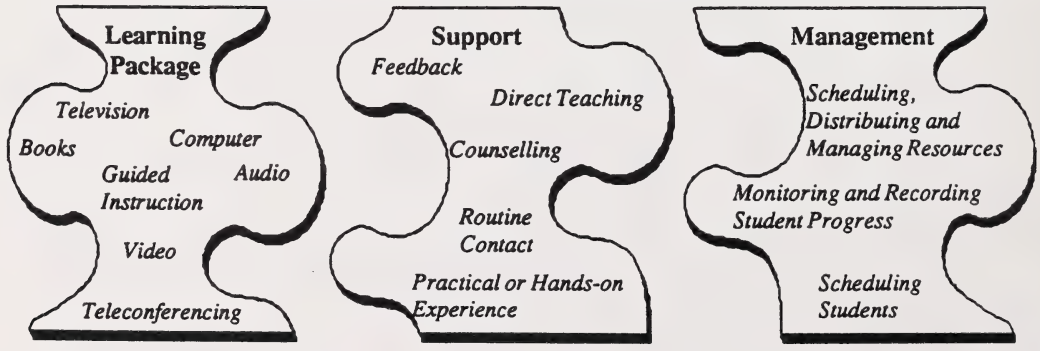
Open-learning is the concept of opening up opportunities by overcoming barriers of time, pace and place by giving the learners a package specially designed to enable them to learn on their own for at least some of the time.

Such a concept is not new. Many teachers can recite attempts to establish an individualized learning system as they recognized the importance of trying to personalize courseware to meet each individual student's needs. But these efforts often failed, due to lack of time and lack of quality materials that conformed to Alberta specifications.

Due to advanced educational technology and improved Alberta-specific learning packages, a student-centered approach is now possible. Improved technology now allows us to provide support to learners, individually, regardless of their pace or location. A teacher cannot be in 28 places at one time offering guidance - media and a well-designed learning package can satisfy individual needs. Technology can also help provide an effective management system needed to track the students as they progress independently through the materials.

The key to a successful open-learning system depends on three vital elements: a learning package specially designed to enable students to learn effectively on their own, for at least some of the time; various kinds of learner support; and a management system and style that ensures that the open-learning system runs smoothly.

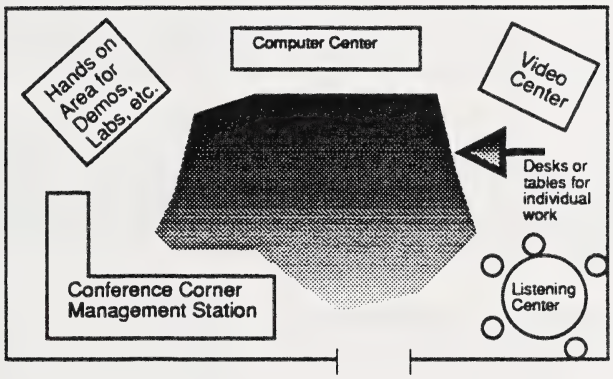
The Key to a Successful Open-Learning System



1. Learning Package

The specially designed learning package needed for a successful open-learning system has been developed for you. The objectives teach current Alberta specifications using strategies designed for individualized instruction. As the learning facilitator, you need to be sure to have all the components in the learning package available to students as needed.

If adequate numbers of media are available to satisfy the demand, a center can be established for specific media:



You may not have the luxury to have enough hardware to set up a permanent video or computer center in your classroom. In that case, students should be encouraged to plan ahead. Perhaps every 3 - 5 days they should preview their materials and project when they would need a certain piece of media. This would allow you to group students, if necessary, or reserve media as required.

2. Support

Support is definitely a key element for any successful learning, and when you're planning an individualized, non-paced program, you need to carefully plan when and how support will be given.

The materials contain a form of consistent support by providing immediate feedback for each of the activities included in the Unit Booklet. High school mathematics units may have solutions, models, explanations and guides included in the Appendix of every Unit Booklet. These are included so students can receive immediate feedback to clarify and reinforce their basic understanding before they move on to higher levels of thinking.

As the learning facilitator, you may be needed to offer more personal guidance to those students having difficulty, or you may need to reinforce the need for doing these activities carefully before attempting the assignments in the Assignment Booklet.

The activities include choices and pathways. If a student is having difficulty, you may need to encourage that student to work on all the choices rather than one. This would provide additional instruction and practice in a variety of ways.

Another form of support is routine contact with each individual. This might be achieved with a bi-weekly conference scheduled by you, or as students reach a certain point (e.g. after each section is completed), they may be directed to come to the conference area.

Special counselling may be needed to help students through difficult stages. Praise and encouragement are important motivators, particularly for those students who are not used to working independently.

Direct teaching may be needed and scheduled at certain points in the program. This might involve small groups or a large group. It might be used to take advantage of something timely (e.g. election, eclipse, etc.) or something prescheduled like the demonstration of a process, or involving students in a hands-on, practical experience.

Support at a distance might include tutoring by phone, teleconferencing, faxing, or planned visits. These contacts are the lifeline between learners and distance education teachers, so a warm dialogue is essential.

3. Management

Good management of an open learning system is essential to the success of the program.
The following areas need action to ensure that the system runs smoothly:

1. Scheduling, Distributing and Managing Resources - As discussed earlier, this may require a need for centers or a system for students to project and reserve the necessary resources.

2. Scheduling Students - Students and teachers should work together to establish goals, course completion timelines and daily timelines. Although students may push to continue for long periods of time (e.g. all morning), teachers should discourage this. Concentration, retention and motivation is improved by taking scheduled breaks.

3. Monitoring Student Progress - You will need to record when units are completed by each student. Your data might also include the projected date of completion if you are using a student contract approach.

Sample of a Student Progress Chart

Mathematics		Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6	Unit 7	Unit 8	Final Test
Billy Adams	P									
	A									
Jane Bell	P									
	A									
Joe Bloom	P									
	A									
P = Projected Completion Date A = Actual Completion Date										

The student could keep a personal log as well.
Such tracking of data could be stored easily on a computer.

4. Recording Student Assessments - You will need to record the marks awarded to each student for work completed in each unit Assignment Booklet. The marks from these assignment booklets will contribute to a portion of the student's final mark. The final test will also be used, and other criteria may also be added (a special project, effort, attitude, etc.). Whatever the criteria is, it should be made clear to all students at the beginning.

Sample of a Student's Assessment Chart

Mathematics	Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6	Unit 7	Unit 8	Years Aver.	Final Test	Final Mark
Billy Adams	67	65	54	47	78	67	63	77	65		
Jane Bell	43	50	54	55	48	42	52	41	43		
Joe Bloom	65	65	66	68	67	70	66	61	66		

Letter grading could easily be substituted.

5. Recording Effectiveness of System - Keep ongoing records of how the system is working. This will help you in future planning.

Sample of a System Assessment Chart

Module 1			
Date	Unit Booklet	Assignment Booklet	Resources-Media

The Role of the Teacher in an Open-Learning Classroom

The teachers in a conventional classroom spend a lot of time talking to large groups of learners. The situation in open learning requires a different emphasis. Teachers will probably meet learners individually or in very small groups.

With this approach it is necessary to move beyond the idea of a passive learner depending largely on the continually supportive teacher. The teacher must aim to build the student's confidence, to stimulate the learner into self-reliance and guide the learner to take advantage of routes that are most meaningful and applicable to the learner.

These materials are student-centered not teacher-centered. The teacher needs to facilitate learning by providing general support to the learner.

Evaluation

Evaluation is important to the development of every learner. Data-gathering, processing, and decision-making, both at the student and teacher level, serve as means of identifying strengths and weaknesses.

These specially-designed learning packages contain many kinds of informal and formal evaluation.

1. Observation

In the classroom the teacher has the opportunity to see each student perform every day and to become aware of the level and nature of each student's performance.

Observations are more useful if they are recorded in an organized system. The following list of questions is a sample of types of observations and how they can be collected.

Observation Checklist

	Students' Names			
1. Does the student approach the work in a positive manner?				
2. Is the student struggling with the reading level?				
3. Does the student make good use of time?				
4. Does the student apply an appropriate study method?				
5. Can the student use references effectively, etc.?				

Observation may suggest a need for an individual interview with a student.

2. Individual Conferences

Individual conferences may be paced (scheduled) by the calendar or at certain points in the unit, or they may be set up only as needed or requested.

During these conferences teachers can determine the student's progress and they can assess the student's attitudes toward the subject, program, school and self, as well as his or her relationship with other students. With guided questions, the teacher can encourage oral self-assessment: the student can discuss his or her own strengths or weaknesses in regard to the particular topic, unit or subject area.

3. Self-Appraisal

Self-appraisal helps students recognize their own strengths and weaknesses. By including activities that require self-assessment, students also gain immediate feedback and clarification at early stages in the learning process. Teachers need to promote a responsible attitude toward these self-assessment activities. Becoming effective self-assessors is a crucial part of becoming an autonomous learner. By instructing, motivating, providing positive reinforcement and supervising systematically, the learning facilitator will help students develop a positive attitude toward their own progress.

For variations, students may be paired and peer-assessing may become part of the system. The teacher may decide to have the student self-assess some of the activities, have a peer assess other activities, and become directly involved in assessing the remainder of the activities.

When the activities have been assessed, the student should be directed to make corrections. This should be made clear to students right from the start. It is important to note the correct association between the question and response to clarify understanding, aid retention and be of use for study purposes.

Many of the activities include choices for students. If the student is having difficulty, more practice may be warranted, and the student may need to be encouraged to do more of the choices.

Each topic within a unit includes additional types of activities called Extra Help and Extensions. Students are expected to be involved in the decision as to which pathway best suits their needs. They may decide to do both.

Self-appraisal techniques can also be introduced at the individual conferences. Such questions as the following might be included:

a. What steps are you taking to improve your understanding of this topic?
b. What method of study do you use most?
c. How do you organize your material to remember it?
d. What steps do you follow when doing an assignment in your Assignment Booklet?
e. What could you do to become an even better reader?
f. Do you have trouble following directions?
g. Did you enjoy this unit?

A chart or checklist could be used for recording responses.

4. Informal Evaluation: Assignments

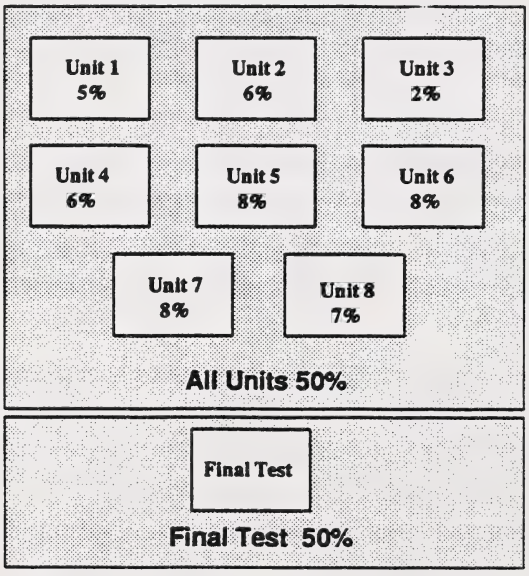
Informal evaluation such as the assignments included in each unit, are an invaluable aid to the teacher. They offer ongoing assessment information about the student's achievement and the behavior and attitudes that affect that achievement.

Each unit contains a separate booklet called the Assignment Booklet. This booklet assesses the knowledge or skills that the student has gained from the module. The student's mark for the module may be based solely on the outcome of learning evident in the Assignment Booklet; however, you may decide to establish a value for other variables such as attitude or effort. It is important that you establish at the beginning which outcomes will be evaluated , and that all students clearly understand what is expected.

5. Final Test

All LFM's include a formal final test which can be photocopied for each member of the class. The test, closely linked to the learning outcomes stated in the unit booklets, gives the teacher precise information concerning what each student can or cannot do. Answers, explanations, and marking guides are also included.

The value of the final test and each unit is the decision of the classroom teacher. Following is a suggestion only.



Introducing Students to the System

Your initiation to these learning materials began with a basic survey of what was included and how the components varied. This same process should be used with the class. After the materials have been explored, a discussion might include what the advantages and the disadvantages might be of learning independently or in small groups. The roles of the students and teacher should be analyzed. The necessary progress checks and rules need to be addressed. Your introduction should motivate students and build a responsible attitude toward learning autonomously.

Skill Level

It is important for students to understand that there are certain skills that they will need in order to deal successfully with the course materials. They are listed below:

1. understanding and using instructional materials (table of contents, index, list of illustrations, appendices, bibliography, glossary)
2. interpreting maps, graphs, charts
3. using reference materials
4. recognizing special symbols
5. using a scientific calculator

Other general skills are using reliable study methods, outlining, and learning to read at a flexible rate.

To decide the level and amount of instruction needed to accommodate the varied levels among students, you may wish to prepare and administer skill inventories or pretests. If most students need help with a particular skill, you may want to plan a total class instructional session. If only certain students lack a skill, you may want to set up a temporary skill group to help students who need it, or you may want to develop a skills file for this purpose.

Reading Level

These course materials are largely print-based, but poorer readers need not be discouraged. It is important that you assure the students that these materials have been designed for easy reading. The authors have employed special strategies that lower and control the reading level. Some of them are

1. the conscious selection of vocabulary and careful structuring of sentences to keep the materials at an independent reading level
2. the integration of activities, examples and illustrations to break text into appropriate-sized chunks
3. the inclusion of many kinds of organizers (advance, graphic, intermediate, concept mapping, post-organizers) to help give students a structure for incorporating new concepts

4. the recognition that vocabulary and concepts are basic to understanding content materials, and thus must be handled systematically (defined in context, through marginal notes, footnotes, and often in a specialized glossary)

5. the acknowledgement that background knowledge and experience plays a vital role in comprehension

6. the systematic inclusion of illustrations and optional videos to help poorer readers and visual learners, and audiocassettes and software also as an alternative to print-based learning

7. a variety of formats (paragraphs, lists, charts, etc.) to help poorer readers who do not absorb or retain main ideas easily in paragraph format

8. the inclusion of media pathways and activity choices to encourage an active rather than passive approach

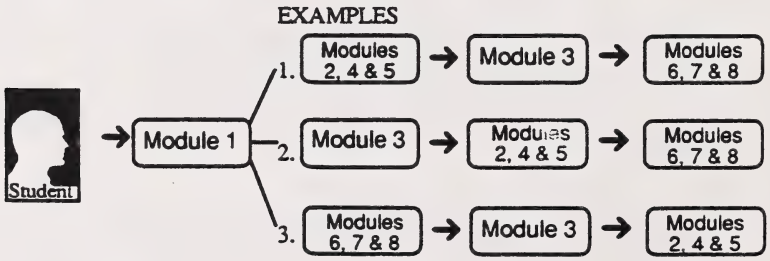
9. instruction in a meaningful setting rather than in a contrived, workbook style.

10. using purposeful reading, viewing and doing to produce better interpretation of the course materials

11. the recognition that students need structured experiences when reading, viewing or listening instructional materials: developing pupil readiness, determining the purpose, providing guided instruction and feedback, rereading if necessary and extending. This structure closely resembles the reading process.

To help make the learning package more readable, you can begin in your unit preparation by reading (viewing, listening) all the related materials that are going to be used. You need a solid background in order to assess and develop a background knowledge for students. The students' experiential bases may be assessed through brainstorming sessions concerning the topic, having students write down all the ideas they have about the topic, or by using visuals and guided questions to predict what the topic might be about.

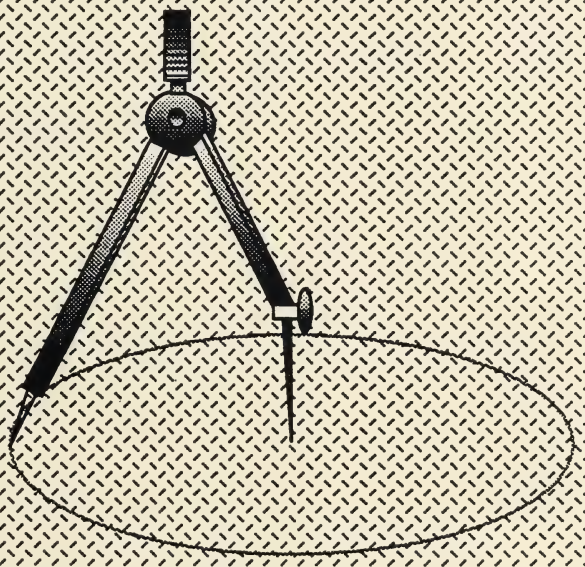
It is recommended that you start with Unit 1 as this unit includes basic information on number systems. In many courses, further units can be tackled in any order. For solutions to all activities check in the Appendix of the respective unit. What will follow will be the solution to the Assignment Booklets, final test and the solutions to the final test.



MATHEMATICS 23

LEARNING FACILITATOR'S MANUAL

UNIT 1



Distance Learning

Alberta EDUCATION

Topic 1: Evaluating Radicals

(2) 1. Complete the following.

a. In the expression $5\sqrt{2a}$, what is the name given to $\sqrt{2a}$?

$\sqrt{2a}$ is called the radical.

b. In the expression $\sqrt{x}$, what is the index?

The index is 2.

c. In the expression $2\left(\sqrt[3]{3b}\right)$, what is the name given to $3b$?

$3b$ is called the radicand.

(5) 2. Evaluate the following, and express your answers to four decimal places.

a. the square root of 711 26.6646

b. the square root of 485 22.0227

c. the square root of 454 21.3073

d. the square root of 904 30.0666

e. the square root of 1364 36.9324

(8) 3. Estimate, to the nearest whole number, the square root of each of the following, and then show the approximate square root of each to four decimal places.

	Estimate (to the nearest whole number)	Square Root
a. $\sqrt{759}$	28	27.5500
b. $\sqrt{261}$	16	16.1555
c. $\sqrt{2148}$	46	46.3465
d. $\sqrt{44\ 034}$	210	209.8428

(3) 4. Evaluate the cube roots of the following. Express your answers to four decimal places.

a. $\sqrt[3]{276} = 6.5108$

b. $\sqrt[3]{415} = 7.4590$

c. $\sqrt[3]{770} = 9.1657$

(6) 5. Estimate the cube roots of the following.

	Estimate (to the nearest whole number)	Approximate Cube Root (to four decimal places)
a. $\sqrt[3]{242}$	6	6.2317
b. $\sqrt[3]{380}$	7	7.2432
c. $\sqrt[3]{135}$	5	5.1299

(6) 6. Evaluate the following.

a. $\sqrt[3]{-\frac{1}{64}} = \sqrt[3]{\left(\frac{-1}{4}\right)^3} = \frac{-1}{4}$

b. $\sqrt{\frac{25}{16}} = \sqrt{\left(\frac{5}{4}\right)^2} = \frac{5}{4}$

c. $\sqrt{6\frac{1}{4}} = \sqrt{\frac{25}{4}} = \sqrt{\left(\frac{5}{2}\right)^2} = \frac{5}{2}$

(6) 7. Evaluate the following.

a. $(-343)^{\frac{1}{3}} = \left[(-7)^3\right]^{\frac{1}{3}} = -7$

b. $(0.008)^{\frac{1}{3}} = \left[(0.2)^3\right]^{\frac{1}{3}} = 0.2$

c. $81^{\frac{1}{2}} = \left(9^2\right)^{\frac{1}{2}} = 9$

Topic 1

_____ marks

Topic 2: Rational Exponents

(8) 1. Simplify the following.

a. $3m^4y^5 \times 2m^2y^2 = (3\times 2)\left(m^{4+2}\right)\left(y^{5+2}\right) = 6m^6y^7$

b. $\frac{4m^5}{2m^2} = \frac{4}{2}\left(m^{5-2}\right) = 2m^3$

c. $\frac{b^{12}}{b^5} = b^{12-5} = b^7$

d. $\frac{a^2b^{-5}}{a^{-3}b^{-7}} = a^{2+3}b^{-5+7} = a^5b^2$

(8) 2. Evaluate the following.

a. $\left(-\frac{3}{4}\right)^3 = \frac{(-3)^3}{4^3} = \frac{-27}{64}$

b. $(-6)^{-3} = \frac{1}{(-6)^3} = \frac{1}{-216}$

c. $4^0 + a = 1 + a$

d. $\frac{2^{-1}+3^{-1}}{4^{-3}} = \frac{\frac{1}{2}+\frac{1}{3}}{\frac{1}{4^3}} = \frac{\frac{3+2}{6}}{\frac{1}{64}} = \frac{\frac{5}{6}}{\frac{1}{64}} = \frac{5}{\cancel{6}_{3}} \times \frac{\overset{32}{\cancel{64}}}{1} = \frac{160}{3} = 53\frac{1}{3}$

(10) 3. Simplify.

a. $\left(c^{7}\right)^{1} = c^{7\times 1} = c^{7}$

b. $\left(m^{7}\right)^{2} = m^{7\times 2} = m^{14}$

c. $\left(c^{2}d^{2}\right)^{3} = c^{2\times 3}d^{2\times 3} = c^{6}d^{6}$

d. $\left(ab^{3}\right)^{-1} = a^{-1}b^{-3} = \frac{1}{ab^{3}}$

e. $\left(\frac{x^{-3}y}{z}\right)^{-2} = \frac{x^{6}y^{-2}}{z^{-2}} = \frac{x^{6}z^{2}}{y^{2}}$

(10) 4. Transform the given exponential expression to its radical equivalent and the given radical expression to its exponential equivalent.

a. $y^{\frac{2}{3}} = \left(\sqrt[3]{y}\right)^{2}$

b. $3^{-\frac{1}{2}} = \left(\sqrt[2]{3}\right)^{-1} = \frac{1}{\sqrt[2]{3}}$

c. $(-12)^{\frac{4}{3}} = \left(\sqrt[3]{-12}\right)^{4}$ or $\sqrt[3]{(-12)^{4}}$

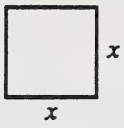
d. $\sqrt[4]{5} = 5^{\frac{1}{4}}$

e. $\sqrt[3]{\left(\frac{3}{4}\right)^{-2}} = \left[\left(\frac{3}{4}\right)^{-2}\right]^{\frac{1}{3}} = \left(\frac{3}{4}\right)^{\frac{-2}{3}}$ or $\left(\frac{4}{3}\right)^{\frac{2}{3}}$

④ 5. The Brown Manufacturing Company was asked to make square lids for food containers. Each lid was to have an area of $0.89\ \text{m}^2$. Find the length of each side. Give your answer to three decimal places.

Let x be the length of the side.

$$\text{Area} = x \times x$$
$$= x^2$$
$$\text{Therefore, } x^2 = 0.89\ \text{m}^2.$$
$$x = \sqrt{0.89\ \text{m}^2}$$
$$x = 0.943\ \text{m}$$



The length of each side is **0.943 m.**

⑥ 6. The radius of a sphere is given by $r = \sqrt[3]{\frac{3V}{4\pi}}$, where V represents the volume. What is the radius of a beach ball if its volume is $530.03\ \text{cm}^3$? Give your answer to three decimal places.

$$r = \sqrt[3]{\frac{3V}{4\pi}}$$
$$r = \sqrt[3]{\frac{3(530.03)}{4 \times \pi}}$$
$$= \sqrt[3]{\frac{1590.09}{4\pi}}$$
$$= \sqrt[3]{126.5353}$$
$$= 5.020$$

The radius of the beach ball is **5.020 cm.**
(Also accept 5.021 cm.)

(6) 7. The radius of a sphere can also be written as $r=\left(\frac{3V}{4\pi}\right)^{\frac{1}{3}}$. Calculate the radius of a dome roof which has a volume of $21\,000\ \text{m}^3$. Assume that the dome roof is half of a sphere. Give your answer to three decimal places.

$$\text{Total volume} = 2\left(21\,000\ \text{m}^2\right)$$

$$= 42\,000\ \text{m}^3$$

$$r=\left(\frac{3V}{4\pi}\right)^{\frac{1}{3}}$$

$$r=\left(\frac{3\times 42\,000}{4\times\pi}\right)^{\frac{1}{3}}$$

$$r=(10\,026.761)^{\frac{1}{3}}$$

$$r=21.564\ \text{m}$$

The radius of the dome roof is **21.564 m.**
(Allow slight variations in the third decimal place. Allow 21.567 m.)

8. The formula $T=2\pi\sqrt{\frac{l}{g}}$ gives the time in seconds for a single swing of a pendulum.

(6) a. Using the above formula, calculate the time it will take for one swing of the pendulum when $l=500$ cm and $g=980\ \text{cm/s}^2$. Give your answer to three decimal places.

$$T=2\pi\sqrt{\frac{l}{g}}$$

$$T=2\times\pi\sqrt{\frac{500}{980}}$$

$$=2\pi\sqrt{0.5102}$$

$$=2\pi\times 0.7143$$

$$=4.488$$

It will take **4.488** seconds for one swing of the pendulum.
(Allow slight variations in the third decimal place. Allow 4.486 s.)

⑥ b. Using the previous formula, calculate the length of the pendulum in metres when $T = 3.4$ s and $g = 9.8\ \text{m/s}^2$. Give your answer to three decimal places.

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$3.4 = 2 \times \pi \sqrt{\frac{l}{9.80}}$$

$$\frac{3.4}{2\pi} = \sqrt{\frac{l}{9.80}}$$

$$\frac{3.4}{2\pi} = \frac{\sqrt{l}}{3.1305}$$

$$\frac{3.4 \times 3.1305}{2\pi} = \sqrt{l}$$

$$1.693\,99 = \sqrt{l}$$

$$2.870 = l$$

The length of the pendulum is **2.870 m.**

(Allow slight variations in the third decimal place. Allow 2.873 m.)

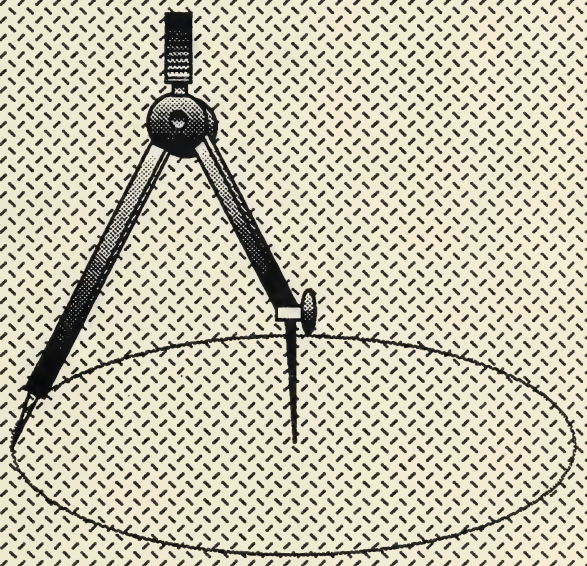
Topic 2

_____ marks

MATHEMATICS 23

LEARNING FACILITATOR'S MANUAL

UNIT 2



Distance Learning

Alberta EDUCATION

Topic 1: Solving and Verifying Linear Equations

(3) 1. Evaluate the polynomial $3x^2-5x+6$ when $x=-4$.

$$\begin{aligned}3(-4)^2-5(-4)+6 &= 3(16)+20+6\\ &= 48+20+6\\ &= 74\end{aligned}$$

(3) 2. Simplify the expression below by adding or subtracting as indicated.

$$(5x^2-6x+3)+(-2x^2+5x-3)-(7x^2-5x-8)$$

$$\begin{aligned}&=5x^2-6x+3-2x^2+5x-3-7x^2+5x+8\\ &=(5x^2-2x^2-7x^2)+(-6x+5x+5x)+(3-3+8)\\ &=-4x^2+4x+8\end{aligned}$$

(4) 3. Simplify each expression below by multiplying.

a. $7x(3x^2-5x+2)$

$$=21x^3-35x^2+14x$$

b. $(2x-3y)(-x+5y)$

$$\begin{aligned}&=-2x^2+10xy+3xy-15y^2\\ &=-2x^2+13xy-15y^2\end{aligned}$$

(2) 4. Solve $\frac{2x}{3}-\frac{x}{5}=\frac{2}{3}$. Verify your answer.

Multiply every term by 15.

$$\cancel{15}^{5}\left(\frac{2x}{\cancel{3}_{1}}\right)-\left(\cancel{15}^{3}\right)\frac{x}{\cancel{5}_{1}}=\left(\cancel{15}^{5}\right)\left(\frac{2}{\cancel{3}_{1}}\right)$$

$$10x-3x=10$$

$$7x=10$$

$$x=\frac{10}{7} \text{ or } 1\frac{3}{7}$$

Verification:

LS	RS
$\frac{2}{3}\left(\frac{10}{7}\right)-\frac{1}{5}\left(\frac{10}{7}\right)$	$\frac{2}{3}$
$\frac{20}{21}-\frac{10}{35}$	
$\frac{20}{21}-\frac{6}{21}$	
$\frac{2}{3}$	

LS = RS

(3) 5. The difference of two numbers is 5. The sum of $\frac{1}{4}$ of the smaller number and $\frac{1}{6}$ of the larger number is $\frac{5}{6}$. Find the two numbers. Let x be the larger number, then $x-5$ must be the smaller number.

$$\frac{x-5}{4}+\frac{x}{6}=\frac{5}{6}$$

Multiply every term by 12.

$$\cancel{12}^{3}\left(\frac{x-5}{\cancel{4}_{1}}\right)+\cancel{12}^{2}\left(\frac{x}{\cancel{6}_{1}}\right)=\cancel{12}^{2}\left(\frac{5}{\cancel{6}_{1}}\right)$$

$$3(x-5)+2(x)=2(5)$$

$$3x-15+2x=10$$

$$5x=25$$

$$x=5$$

and $x-5=0$

Therefore, the two numbers are 0 and 5.

(2) 6. Solve each equation below.

a. $4x-3(x-7)=2(x+4)$

$$\begin{aligned} 4x-3x+21&=2x+8\\ x+21&=2x+8\\ 21&=x+8\\ 13&=x \end{aligned}$$

(3) b. $\frac{2}{3}(x-7)-\frac{5}{2}(-3x+1)=\frac{3}{4}(7-5x)$

$$\begin{aligned} \cancel{12}^{4}\left(\frac{2}{\cancel{3}_{1}}(x-7)\right)-\cancel{12}^{6}\left(\frac{5}{\cancel{2}_{1}}(-3x+1)\right)&=\cancel{12}^{3}\left(\frac{3}{\cancel{4}_{1}}(7-5x)\right)\\ 4[2(x-7)]-6[5(-3x+1)]&=3[3(7-5x)]\\ 8(x-7)-30(-3x+1)&=9(7-5x)\\ 8x-56+90x-30&=63-45x\\ 98x-86&=63-45x\\ 143x-86&=63\\ 143x&=149\\ x&=\frac{149}{143} \end{aligned}$$

(2) 7. Translate each of the following into a mathematical phrase.

a. Four less than three times x is 5.

b. Half of the sum of a certain number and 5 is added to $\frac{1}{3}$ of the difference of the number from 4.

a. $3x-4=5$

b. $\frac{1}{2}(x+5)+\frac{1}{3}(4-x)$

(3) 8. When 63 is added to a certain number, the result is 8 times the number, decreased by 42. What is the number?

$$\begin{aligned} x+63 &= 8x-42 \\ 63 &= 7x-42 \\ 105 &= 7x \\ \frac{105}{7} &= x \\ 15 &= x \end{aligned}$$

The number is 15.

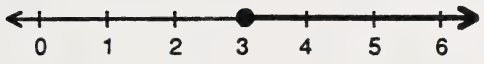
Topic 1

_____ marks

Topic 2: Solving Linear Inequalities and Graphing Their Solutions

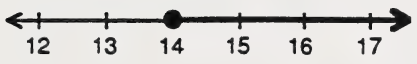
(4) 1. Solve and graph $2(x-5)+3(x+2)\geq 11$.

$$\begin{aligned} 2x-10+3x+6 &\geq 11 \\ 5x-4 &\geq 11 \\ 5x &\geq 15 \\ x &\geq 3 \end{aligned}$$



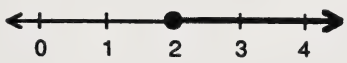
(4) 2. Solve and graph $x-3\geq\frac{2x+5}{3}$.

$$\begin{aligned} 3x-9 &\geq 2x+5 \\ x &\geq 14 \end{aligned}$$



(4) 3. Solve and graph $\frac{2-x}{3}\leq\frac{x-2}{2}$.

$$\begin{aligned} \overset{2}{\cancel{6}}\left(\frac{2-x}{\cancel{3}_1}\right) &\leq \left(\frac{x-2}{\cancel{2}_1}\right)\overset{3}{\cancel{6}} \qquad \text{Multiply each term by 6.} \\ 2(2-x) &\leq 3(x-2) \\ 4-2x &\leq 3x-6 \\ -5x &\leq -10 \\ x &\geq 2 \end{aligned}$$



(3) 4. Solve the following inequality.

$$6-0.3x<4+1.7x$$

$$6-4<1.7x+0.3x$$
$$2<2x$$
$$1<x$$
$$\text{or } x>1$$

(3) 5. Solve the following inequality.

$$\frac{5-x}{2}+\frac{x}{4}<\frac{x+2}{4}$$

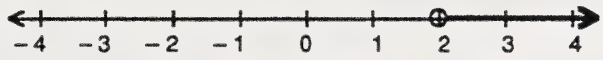
$$\frac{5-x}{2}+\frac{x}{4}<\frac{x+2}{4}$$
$$2(5-x)+x<x+2$$
$$10-2x+x<x+2$$
$$10-x<x+2$$
$$10<2x+2$$
$$8<2x$$
$$4<x \text{ or } x>4$$

(3) 6. Solve the following inequality.

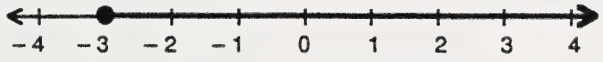
$$y+\frac{3}{4}(y-4)\geq\frac{2}{3}(y+2)$$

$$12(y)+12\left[\frac{3}{4}(y-4)\right]\geq 12\left[\frac{2}{3}(y+2)\right]$$
$$12y+9(y-4)\geq 8(y+2)$$
$$12y+9y-36\geq 8y+16$$
$$21y-36\geq 8y+16$$
$$21y\geq 8y+52$$
$$13y\geq 52$$
$$y\geq 4$$

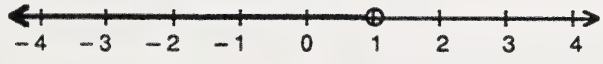
(4) 7. Write an inequation that describes each of the following graphs.

a. 

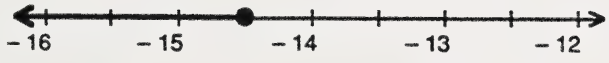
$x > 2$

b. 

$x \geq -3$

c. 

$x < 1$

d. 

$x \leq -14\frac{1}{2}$

Topic 2

_____ marks

Topic 3: Factoring Polynomials of the Form $ax^2+bx+c;\ a,b,c \in I$

(3) 1. Factor each of the following completely.

a. x^3-9x

$$x^3-9x = x\left(x^2-9\right)$$
$$= x(x+3)(x-3)$$

(3) b. $25x^2-30x+9$

$$25x^2-30x+9 = (5x)^2-2(5x)(3)+3^2$$
$$= (5x-3)^2$$

(3) c. $8x^4-50$

$$8x^4-50 = 2\left(4x^4-25\right)$$
$$= 2\left(2x^2+5\right)\left(2x^2-5\right)$$

③ d. $3x^3y^2+3x^2y^2-18xy^2$

$$3x^3y^2+3x^2y^2-18xy^2 = 3xy^2\left(x^2+x-6\right)$$
$$= 3xy^2(x+3)(x-2)$$

③ e. $x^2-2x-63$

$$x^2-2x-63=(x-9)(x+7)$$

③ f. $48b^4-3$

$$48b^4-3=3\left(16b^4-1\right)$$
$$=3\left(4b^2+1\right)\left(4b^2-1\right)$$
$$=3\left(4b^2+1\right)(2b+1)(2b-1)$$

③ g. x^2-y^2+4y-4

$$\begin{aligned} x^2-y^2+4y-4 &= x^2-\left(y^2-4y+4\right) \\ &= x^2-(y-2)^2 \\ &= (x-y+2)(x+y-2) \end{aligned}$$

④ h. $(4y-6)^2-(4y+6)^2$

$$\begin{aligned} (4y-6)^2-(4y+6)^2 &= (4y-6+4y+6)\left(4y-6-[4y+6]\right) \\ &= (8y)(4y-6-4y-6) \\ &= (8y)(-12) \\ &= -96y \end{aligned}$$

Topic 3

_____ marks

Topic 4: Solving Simple Quadratic Equations

③ 1. Solve $x^2-12x+35=0$.

$$(x-7)(x-5)=0$$
$$x-7=0 \text{ or } x-5=0$$
$$x=7 \qquad x=5$$

③ 2. Solve $28x^2-112=0$.

$$28x^2=112$$
$$x^2=4$$
$$x=\pm\sqrt{4}$$
$$x=\pm 2$$

④ 3. Solve and verify $(x-2)^2-3=2(x-4)$.

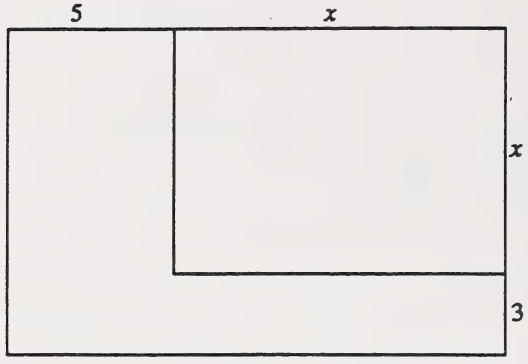
$$x^2-4x+4-3=2x-8$$
$$x^2-6x+9=0$$
$$(x-3)^2=0$$
$$x-3=0$$
$$x=3$$

Verify $x=3$.

LS	RS
$(3-2)^2-3$	$2(3-4)$
1^2-3	$2(-1)$
-2	-2

LS = RS

④ 4. The area of the large rectangle below is 48 cm ^{2}. Find x.



width × length = Area

$(x+3)(x+5)=48$

$x^2+8x+15=48$

$x^2+8x-33=0$

$(x+11)(x-3)=0$

$x+11=0$ or $x-3=0$

$x=-11$ $\quad x=3$

(Discard – 11 since a side of a rectangle cannot be negative.)

x is 3.

(4) 5. The sum of the square of two consecutive numbers is 545. What are the two numbers?

Let the two consecutive numbers be x and $x + 1$.

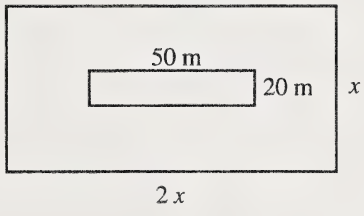
$$x^2 + (x+1)^2 = 545$$
$$x^2 + x^2 + 2x + 1 = 545$$
$$2x^2 + 2x - 544 = 0$$
$$x^2 + x - 272 = 0$$
$$(x+17)(x-16) = 0$$
$$x + 17 = 0 \qquad x - 16 = 0$$
$$x = -17 \qquad x = 16$$

If $x = -17$, then $x + 1 = -16$.

If $x = 16$, then $x + 1 = 17$.

The two numbers are –16 and –17 or 16 and 17.

(4) 6. A diagram of a swimming pool and the surrounding lawn is shown below. If the area of the lawn is four times the area of the pool, what are the dimensions of the lawn? (The area of the lawn and the pool is five times the area of the pool.)



If the area of the lawn is four times the area of the pool, then the area of the lawn and pool is five times the area of the pool.

$$5 \times 50 \times 20 = 2x \times x$$
$$5000 = 2x^2$$
$$0 = 2x^2 - 5000$$
$$0 = x^2 - 2500$$
$$0 = (x+50)(x-50)$$
$$0 = x + 50 \qquad 0 = x - 50$$
$$-50 = x \qquad 50 = x$$
$$\uparrow \qquad 2x = 2 \times 50$$
$$\text{inadmissible} \qquad = 100$$

The dimensions of the lawn are 100 m by 50 m.

③ 7. Solve $(3x-4)^2-(3x+4)^2=240$.

$$
\begin{aligned}
(3x-4)^2-(3x+4)^2 &= 240 \\
(3x-4+3x+4)(3x-4-[3x+4]) &= 240 \\
(6x)(3x-4-3x-4) &= 240 \\
(6x)(-8) &= 240 \\
-48x &= 240 \\
x &= -5
\end{aligned}
$$

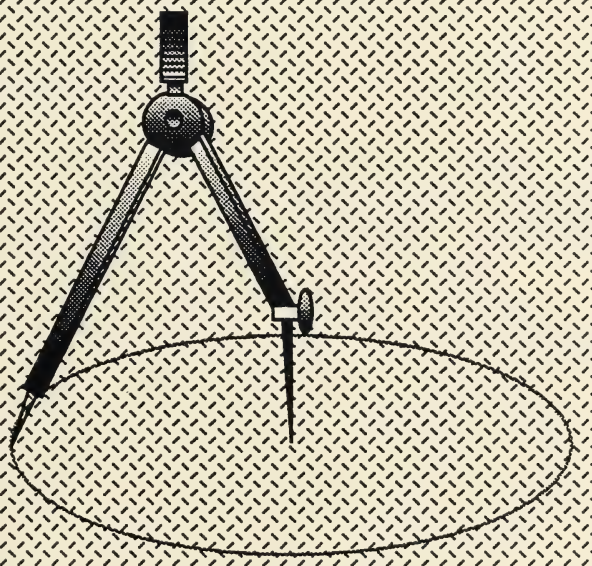
Topic 4

_____ marks

MATHEMATICS 23

LEARNING FACILITATOR'S MANUAL

UNIT 3



Distance Learning

Alberta EDUCATION

Topic 1: Compound Interest

(2) 1. Define each of the following variables from the compound interest formula.

A - amount (principal and interest)

P - original principal

i - interest rate per compounding period

n - number of compounding periods

(8) 2. Complete the table shown below.

Annual Rate of Interest	Compounding Period	Number of Years	Number of Compounding Periods (n)	Rate per Compounding Period (i)
26%	monthly	4	$12 \times 4 = 48$	$\frac{26}{12} = 2.1667\%$
14%	semiannually	12	$2 \times 12 = 24$	$\frac{14}{2} = 7\%$
$8\frac{1}{2}\%$	quarterly	3	$4 \times 3 = 12$	$\frac{8.5}{4} = 2.125\%$
17%	annually	8	$1 \times 8 = 8$	$\frac{17}{1} = 17\%$
6%	daily	1	$365 \times 1 = 365$	$\frac{6}{365} = 0.016\ 44\%$

(8) 3. John Rau wants to invest $5000 in a Guaranteed Investment Certificate for eighteen months. The G.I.C. pays interest at a rate of 9%/a compounded semiannually. Calculate the total amount of interest John will earn on the investment in the table provided, using repeated simple interest calculations.

6-month Term	Principal Plus Previous Interest	Simple Interest per Term
1st	$5000	$I = \$5000 \times 0.09 \times \frac{6}{12}$ $= \$225.00$
2nd	$5000 + $225 = $5225.00	$I = \$5225 \times 0.09 \times \frac{6}{12}$ $= \$235.13$
3rd	$5225 + $235.13 = $5460.13	$I = \$5460.13 \times 0.09 \times \frac{6}{12}$ $= \$245.71$

Total amount of interest = $225 + $235.13 + $245.71 = $705.84

(3) 4. Mary Noble has built up an overdue balance of $1020.00 using her bank credit card. Mary has stopped using her card and hopes to be able to pay off the balance in four months. The credit card company charges 27%/a compounded monthly on any overdue balance.

a. Identify the values for each of the following:

$P = \$1020$

$i = \frac{27\%}{12}$

$= 2.25\%$

$= 0.0225$

$n = 4$

(3) b. Determine the total amount which must be repaid after four months.

$A = \$1020(1+0.0225)^4$

$= \$1020(1.093\ 083\ 319)$

$= \$1114.94$

Mary would owe the credit card company $1114.94 in four months.

(10) 5. Find the compound interest for the first two compounding periods by using the simple interest formula.

a. $P = \$940$ $r = 12\%$/a compounded monthly

$$I = Prt$$

$$I_1 = \$940 \times 0.12 \times \frac{1}{12}$$

$$= \$9.40$$

$$P = \$940 + \$9.40$$

$$= \$949.40$$

$$I_2 = \$949.40 \times 0.12 \times \frac{1}{12}$$

$$= \$9.49$$

$$\text{Interest} = \$9.40 + \$9.49$$

$$= \$18.89$$

b. $P = \$3800$ $r = 10\%$/a compounded quarterly

$$I_1 = \$3800 \times 0.10 \times \frac{3}{12}$$

$$= \$95.00$$

$$P = \$3800 + \$95$$

$$= \$3895.00$$

$$I_2 = \$3895 \times 0.10 \times \frac{3}{12}$$

$$= \$97.38$$

$$\text{Interest} = \$95 + \$97.38$$

$$= \$192.38$$

Compound Interest

n	3.5%	4%	4.5%	5%	5.5%	6%	6.5%	7%	7.5%	8%
1	1.035 000	1.040 000	1.045 000	1.050 000	1.055 000	1.060 000	1.065 000	1.070 000	1.075 000	1.080 000
2	1.071 225	1.081 600	1.092 025	1.102 500	1.113 025	1.123 600	1.134 225	1.144 900	1.155 625	1.166 400
3	1.108 718	1.124 864	1.141 166	1.157 625	1.174 241	1.191 016	1.207 950	1.225 043	1.242 297	1.259 712
4	1.147 523	1.169 859	1.192 519	1.215 506	1.238 825	1.262 477	1.286 466	1.310 796	1.335 469	1.360 489
5	1.187 686	1.216 653	1.246 182	1.276 282	1.306 960	1.338 226	1.370 087	1.402 552	1.435 629	1.469 328
6	1.229 255	1.265 319	1.302 260	1.340 096	1.378 843	1.418 519	1.459 142	1.500 730	1.543 302	1.586 874
7	1.272 279	1.315 932	1.360 862	1.407 100	1.454 679	1.503 630	1.553 987	1.605 781	1.659 049	1.713 824
8	1.316 809	1.368 569	1.422 101	1.477 455	1.534 687	1.593 848	1.654 996	1.718 186	1.783 478	1.850 930
9	1.362 897	1.423 312	1.486 095	1.551 328	1.619 094	1.689 479	1.762 570	1.838 459	1.917 239	1.999 005
10	1.410 599	1.480 244	1.552 969	1.628 895	1.708 144	1.790 848	1.877 137	1.967 151	2.061 032	2.158 925
11	1.459 970	1.539 454	1.622 853	1.710 339	1.802 092	1.898 299	1.999 151	2.104 852	2.215 609	2.331 639
12	1.511 069	1.601 032	1.695 881	1.795 856	1.901 207	2.012 196	2.129 096	2.252 192	2.381 780	2.518 170
13	1.563 956	1.665 074	1.772 196	1.885 649	2.005 774	2.132 928	2.267 487	2.409 845	2.560 413	2.719 624
14	1.618 695	1.731 676	1.851 945	1.979 932	2.116 091	2.260 904	2.414 874	2.578 534	2.752 444	2.937 194
15	1.675 349	1.800 944	1.935 282	2.078 928	2.232 476	2.396 558	2.571 841	2.759 032	2.958 877	3.172 169
16	1.733 986	1.872 981	2.022 370	2.182 875	2.355 263	2.540 352	2.739 011	2.952 164	3.180 793	3.425 943
17	1.794 676	1.947 900	2.113 377	2.292 018	2.484 802	2.692 773	2.917 046	3.158 815	3.419 353	3.700 018
18	1.857 489	2.025 817	2.208 479	2.406 619	2.621 466	2.854 339	3.106 654	3.379 932	3.675 804	3.996 019
19	1.922 501	2.106 849	2.307 860	2.526 950	2.765 647	3.025 600	3.308 587	3.616 528	3.951 489	4.315 701
20	1.989 789	2.191 123	2.411 714	2.653 298	2.917 757	3.207 135	3.523 645	3.869 684	4.247 851	4.660 957
21	2.059 431	2.278 768	2.520 241	2.785 963	3.078 234	3.399 564	3.752 682	4.410 562	4.566 440	5.033 834
22	2.131 512	2.369 919	2.633 652	2.925 261	3.247 537	3.603 537	3.996 606	4.430 402	4.908 923	5.436 540
23	2.206 114	2.464 716	2.752 166	3.071 524	3.426 152	3.819 750	4.256 386	4.740 530	5.277 092	5.871 464
24	2.283 328	2.563 304	2.876 014	3.225 100	3.614 590	4.048 935	4.533 051	5.072 367	5.672 874	6.341 181
25	2.363 245	2.665 836	3.005 434	3.386 355	3.813 392	4.291 871	4.827 699	5.427 433	6.098 340	6.848 475
26	2.445 959	2.772 470	3.140 679	3.555 673	4.023 129	4.549 383	5.141 500	5.807 353	6.555 715	7.396 353
27	2.531 567	2.883 369	3.282 010	3.733 456	4.244 401	4.822 346	5.475 697	6.213 868	7.047 394	7.988 061
28	2.620 172	2.998 703	3.429 700	3.920 129	4.477 843	5.111 687	5.831 617	6.648 838	7.575 948	8.627 106
29	2.711 878	3.118 651	3.584 036	4.116 136	4.724 124	5.418 388	6.210 672	7.114 257	8.144 144	9.317 275
30	2.806 794	3.243 398	3.745 318	4.321 942	4.983 951	5.743 491	6.614 366	7.612 255	8.754 955	10.062 657
31	2.905 031	3.373 133	3.913 857	4.538 039	5.258 069	6.088 101	7.044 300	8.145 113	9.411 577	10.867 669
32	3.006 708	3.508 059	4.089 981	4.764 941	5.547 262	6.453 387	7.502 179	8.715 271	10.117 445	11.737 083
33	3.111 942	3.648 381	4.274 030	5.003 189	5.852 362	6.840 590	7.989 821	9.325 340	10.876 253	12.676 050
34	3.220 860	3.794 316	4.466 362	5.253 348	6.174 242	7.251 025	8.590 159	9.978 114	11.691 972	13.690 134
35	3.333 590	3.946 089	4.667 348	5.516 015	6.513 825	7.686 087	9.062 255	10.676 581	12.568 870	14.785 344
36	3.450 266	4.103 933	4.877 378	5.791 816	6.872 085	8.147 252	9.651 301	11.423 942	13.511 536	15.968 172
37	3.571 025	4.268 090	5.096 860	6.081 407	7.250 050	8.636 087	10.278 636	12.223 618	14.524 901	17.245 626
38	3.696 011	4.438 813	5.326 219	6.385 477	7.648 803	9.154 252	10.946 747	13.079 271	15.614 268	18.625 276
39	3.825 372	4.616 366	5.565 899	6.704 751	8.069 487	9.703 507	11.658 286	13.994 820	16.785 339	20.115 298
40	3.959 260	4.801 021	5.816 365	7.039 989	8.513 309	10.285 718	12.416 075	14.974 458	18.044 239	21.724 521

(10) 6. Use the compound interest table shown above to determine the value for $(1+i)^n$ given the following values for i and n.

a. $i = 0.055$, $n = 16$, $(1+i)^n =$ **2.355 263**

b. $i = 0.08$, $n = 37$, $(1+i)^n =$ **17.245 626**

c. $i = 0.035$, $n = 25$, $(1+i)^n =$ **2.363 245**

d. $i = 0.07$, $n = 9$, $(1+i)^n =$ **1.838 459**

e. $i = 0.04$, $n = 40$, $(1+i)^n =$ **4.801 021**

(10) 7. Complete the following table using the values for $(1+i)^n$ in question 3.

Value for $(1+i)^n$	Principal	Amount to be Repaid	Amount of Interest Paid
3.a	$ 360	**$847.89**	**$487.89**
3.b	$ 1940	**$33 456.51**	**$31 516.51**
3.c	$ 570	**$1347.05**	**$777.05**
3.d	$ 2100	**$3860.76**	**$1760.76**
3.e	$ 880	**$4224.90**	**$3344.90**

8. Heather wants to invest money in Canada Saving Bonds which mature after seven years. She can determine the amount of the bonds at maturity by using the formula $A = \$1500(1+0.0075)^{84}$. Determine the following.

(1) a. principal

$1500

(2) b. compounding period

$$\frac{7a \times 12m/a}{84} = \frac{84\ m}{84} = 1\ m$$

Compounding is monthly.

(2) c. interest rate per annum

$0.0075 \times 12 = 0.09$
$0.09 \times 100\% = 9\%$

(3) d. value of the Canada Savings Bonds after seven years

$A = \$1500(1+0.0075)^{84}$
$= \$2809.80$

Heather's bonds would be worth $2809.80 after seven years.

④ 9. Ian borrowed $7200 from Capital Trust to purchase a new car. Use the compound interest formula to find the amount of interest Ian must pay after one year if Capital Trust charges 16%/a compounded monthly.

$$A = ? \quad P = \$7200 \quad i = \frac{16\%}{12} \quad n = 1 \times 12$$
$$= 1.3333\% \quad = 12$$
$$= 0.01333$$

$$A = \$7200(1+0.01333)^{12}$$
$$= \$8440.02.$$

$$\text{Interest} = \text{amount} - \text{principal}$$
$$= \$8440.02 - \$7200.00$$
$$= \$1240.02$$

Ian must pay $1240.02.

⑦ 10. Samantha borrowed $15 700 from Total Finance at 17%/a compounded semiannually. She immediately invested this money in an investment which pays 19%/a compounded monthly. How much profit will Samantha make in one year?

$$\text{Borrowed } A = \$15\ 700(1+0.085)^{2}$$
$$= \$18\ 482.43$$
$$A = \$15\ 700(1+0.015\ 833)^{12}$$
$$= \$18\ 956.91$$
$$\text{Profit} = \$18\ 956.91 - \$18\ 482.43$$
$$= \$474.48 \text{ (permit to } \$474.40)$$

Samantha made $474.48 in one year.

(10) 11. Ken invested $80 000 in Barrhead Syndicate.

a. Determine the amount that Ken earns after seven years, given the following interest rates.

i. 9%/a compounded annually

$$A = ? \quad P = \$80\,000.00 \quad i = \frac{9\%}{1} \quad n = 1 \times 7$$
$$= 9\% \qquad = 7$$
$$= 0.09$$

$$A = \$80\,000\,(1+0.09)^{7}$$
$$= \$146\,243.13$$

ii. 8%/a compounded monthly

$$A = ? \quad P = \$80\,000.00 \quad i = \frac{8\%}{12} \qquad n = 12 \times 7$$
$$= 0.006\,667 \qquad = 84$$

$$A = \$80\,000\,(1+0.006\,667)^{84}$$
$$= \$\,139\,797.65$$

(permit to $139 793.76)

iii. $8\frac{1}{2}$%/a compounded semiannually

$$P = \$80\,000 \quad i = \frac{8.5\%}{2} \qquad n = 2 \times 7$$
$$= 0.0425 \qquad = 14$$

$$A = \$80\,000\,(1+0.0425)^{14}$$
$$= \$143\,269.87$$

b. Which is the best interest rate?

The interest rate 9%/a compounded annually is the best interest rate, since this rate provides the greatest return.

When you answer the following questions, use this compound interest table.

Compound Interest

n	3.5%	4%	4.5%	5%	5.5%	6%	6.5%	7%	7.5%	8%
1	1.035 000	1.040 000	1.045 000	1.050 000	1.055 000	1.060 000	1.065 000	1.070 000	1.075 000	1.080 000
2	1.071 225	1.081 600	1.092 025	1.102 500	1.113 025	1.123 600	1.134 225	1.144 900	1.155 625	1.166 400
3	1.108 718	1.124 864	1.141 166	1.157 625	1.174 241	1.191 016	1.207 950	1.225 043	1.242 297	1.259 712
4	1.147 523	1.169 859	1.192 519	1.215 506	1.238 825	1.262 477	1.286 466	1.310 796	1.335 469	1.360 489
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6	1.229 255	1.265 319	1.302 260	1.340 096	1.378 843	1.418 519	1.459 142	1.500 730	1.543 302	1.586 874
7	1.272 279	1.315 932	1.360 862	1.407 100	1.454 679	1.503 630	1.553 987	1.605 781	1.659 049	1.713 824
8	1.316 809	1.368 569	1.422 101	1.477 455	1.534 687	1.593 848	1.654 996	1.718 186	1.783 478	1.850 930
9	1.362 897	1.423 312	1.486 095	1.551 328	1.619 094	1.689 479	1.762 570	1.838 459	1.917 239	1.999 005
10	1.410 599	1.480 244	1.552 969	1.628 895	1.708 144	1.790 848	1.877 137	1.967 151	2.061 032	2.158 925
11	1.459 970	1.539 454	1.622 853	1.710 339	1.802 092	1.898 299	1.999 151	2.104 852	2.215 609	2.331 639
12	1.511 069	1.601 032	1.695 881	1.795 856	1.901 207	2.012 196	2.129 096	2.252 192	2.381 780	2.518 170
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17	1.794 676	1.947 900	2.113 377	2.292 018	2.484 802	2.692 773	2.917 046	3.158 815	3.419 353	3.700 018
18	1.857 489	2.025 817	2.208 479	2.406 619	2.621 466	2.854 339	3.106 654	3.379 932	3.675 804	3.996 019
19	1.922 501	2.106 849	2.307 860	2.526 950	2.765 647	3.025 600	3.308 587	3.616 528	3.951 489	4.315 701
20	1.989 789	2.191 123	2.411 714	2.653 298	2.917 757	3.207 135	3.523 645	3.869 684	4.247 851	4.660 957
21	2.059 431	2.278 768	2.520 241	2.785 963	3.078 234	3.399 564	3.752 682	4.410 562	4.566 440	5.033 834
22	2.131 512	2.369 919	2.633 652	2.925 261	3.247 537	3.603 537	3.996 606	4.430 402	4.908 923	5.436 540
23	2.206 114	2.464 716	2.752 166	3.071 524	3.426 152	3.819 750	4.256 386	4.740 530	5.277 092	5.871 464
24	2.283 328	2.563 304	2.876 014	3.225 100	3.614 590	4.048 935	4.533 051	5.072 367	5.672 874	6.341 181
25	2.363 245	2.665 836	3.005 434	3.386 355	3.813 392	4.291 871	4.827 699	5.427 433	6.098 340	6.848 475
26	2.445 959	2.772 470	3.140 679	3.555 673	4.023 129	4.549 383	5.141 500	5.807 353	6.555 715	7.396 353
27	2.531 567	2.883 369	3.282 010	3.733 456	4.244 401	4.822 346	5.475 697	6.213 868	7.047 394	7.988 061
28	2.620 172	2.998 703	3.429 700	3.920 129	4.477 843	5.111 687	5.831 617	6.648 838	7.575 948	8.627 106
29	2.711 878	3.118 651	3.584 036	4.116 136	4.724 124	5.418 388	6.210 672	7.114 257	8.144 144	9.317 275
30	2.806 794	3.243 398	3.745 318	4.321 942	4.983 951	5.743 491	6.614 366	7.612 255	8.754 955	10.062 657
31	2.905 031	3.373 133	3.913 857	4.538 039	5.258 069	6.088 101	7.044 300	8.145 113	9.411 577	10.867 669
32	3.006 708	3.508 059	4.089 981	4.764 941	5.547 262	6.453 387	7.502 179	8.715 271	10.117 445	11.737 083
33	3.111 942	3.648 381	4.274 030	5.003 189	5.852 362	6.840 590	7.989 821	9.325 340	10.876 253	12.676 050
34	3.220 860	3.794 316	4.466 362	5.253 348	6.174 242	7.251 025	8.590 159	9.978 114	11.691 972	13.690 134
35	3.333 590	3.946 089	4.667 348	5.516 015	6.513 825	7.686 087	9.062 255	10.676 581	12.568 870	14.785 344
36	3.450 266	4.103 933	4.877 378	5.791 816	6.872 085	8.147 252	9.651 301	11.423 942	13.511 536	15.968 172
37	3.571 025	4.268 090	5.096 860	6.081 407	7.250 050	8.636 087	10.278 636	12.223 618	14.524 901	17.245 626
38	3.696 011	4.438 813	5.326 219	6.385 477	7.648 803	9.154 252	10.946 747	13.079 271	15.614 268	18.625 276
39	3.825 372	4.616 366	5.565 899	6.704 751	8.069 487	9.703 507	11.658 286	13.994 820	16.785 339	20.115 298
40	3.959 260	4.801 021	5.816 365	7.039 989	8.513 309	10.285 718	12.416 075	14.974 458	18.044 239	21.724 521

12. Ester Holiday invested $600 in a five-year term deposit. The local bank offered to pay 11%/a compounded semiannually for the five-year term. Use the compound interest table on page 10.

(4) a. How much would Ester receive after five years?

$$P = \$600 \qquad i = \frac{11\%}{2} \qquad n = 2 \times 5$$
$$= 5.5\% \qquad = 10$$

$$A = \$600(1.708\ 144)$$
$$= \$1024.89$$

Ester would receive $1024.89 after five years.

(6) b. After three years, Ester decides to withdraw her money from the term deposit. The bank penalizes her $80 for the early withdrawal of funds. How much money did Ester receive after three years?

$$P = \$600 \qquad i = \frac{11\%}{2} \qquad n = 2 \times 3$$
$$= 5.5\% \qquad = 6$$

$$A = \$600(1.378\ 843) - \$80$$
$$= \$827.31 - \$80$$
$$= \$747.31$$

Ester would receive $747.31 after three years.

13. Use your calculator for this question. Jacob Winslow deposited $3000 in a savings account at his local trust company. The account pays interest at a rate of 9%/a compounded monthly. Jacob withdraws $200 from this account every three months. Determine the amount in Jacob's account after the following periods of time.

(3) a. two months

$$P = \$3000 \qquad i = \frac{9\%}{12} = 0.75\% \qquad n = 1 \times 2 = 2$$

$$\begin{aligned} A &= \$3000(1+0.0075)^2 \\ &= \$3000(1.015\ 056) \\ &= \$3045.17 \end{aligned}$$

Jacob has $3045.17 in his account after two months.

(4) b. three months

$$P = \$3000 \qquad i = \frac{9\%}{12} = 0.75\% \qquad n = 1 \times 3 = 3$$

$$\begin{aligned} A &= \$3000(1+0.0075)^3 - \$200 \\ &= \$3000(1.002\,669) - \$200 \\ &= \$3068.01 - \$200 \\ &= \$2868.01 \end{aligned}$$

Jacob has $2868.01 in his account after three months.

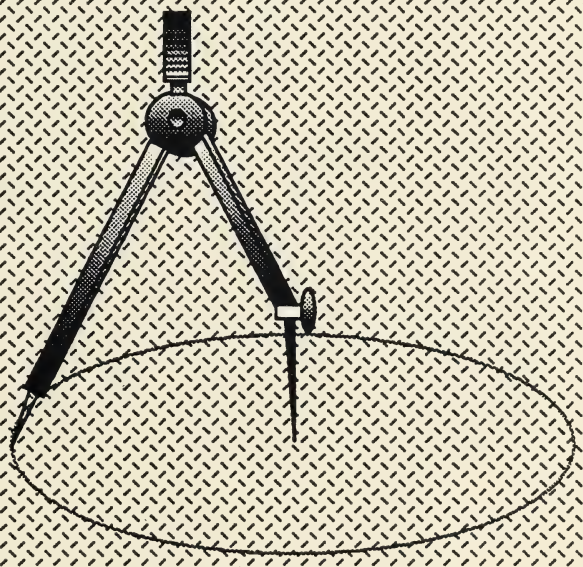
Topic 1

_____ marks

MATHEMATICS 23

LEARNING FACILITATOR'S MANUAL

UNIT 4



Distance Learning

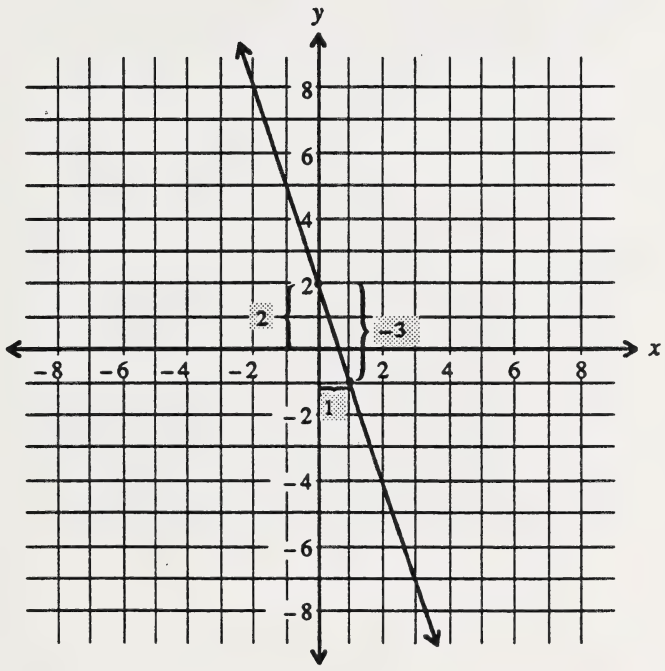
Alberta EDUCATION

Topic 1: Equations and Graphs of Linear Relations

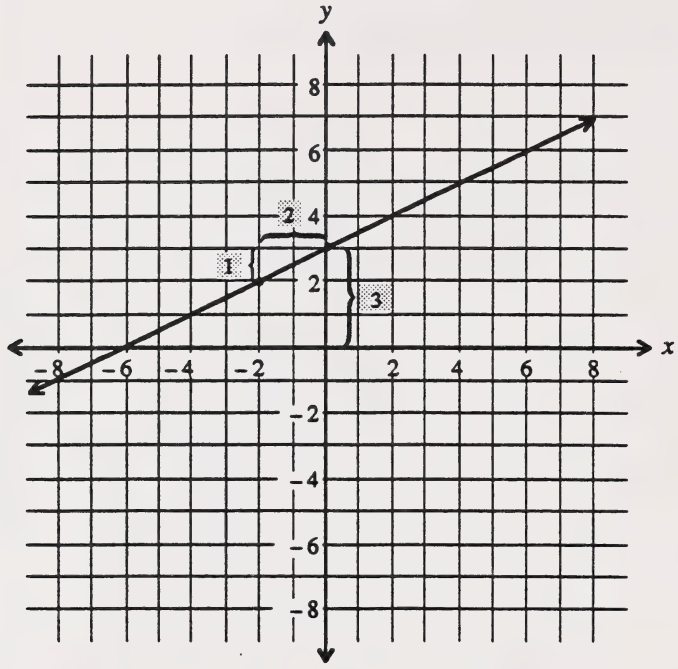
④

1. Draw the graphs for the following conditions.

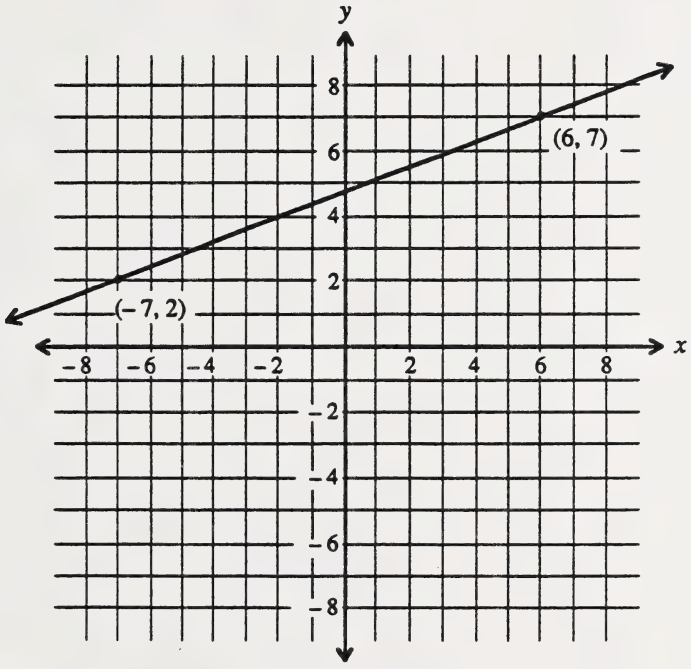
   a. slope of – 3 and *y*-intercept of 2



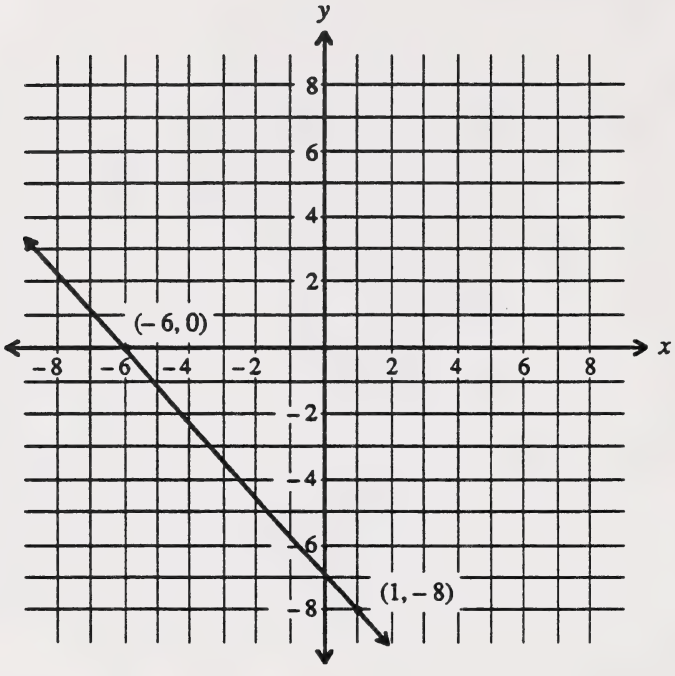
④ b. slope of $\frac{1}{2}$ and y-intercept of 3



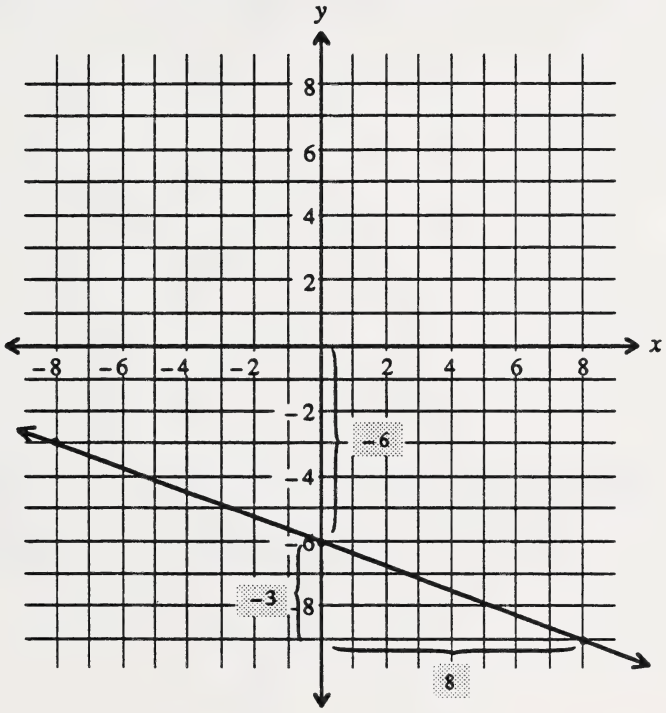
④ c. Two points on the graph are (– 7, 2) and (6, 7).



④ d. Two points on the graph are (– 6, 0) and (1, – 8).



④ e. y-intercept of -6 and slope $-\frac{3}{8}$



(6) 3. a. The graph of a linear relation has a slope of $\frac{7}{8}$ and a y-intercept of $\frac{1}{4}$. Write the equation for the relation.

$m=\frac{7}{8}$ and $b=\frac{1}{4}$ in $y=mx+b$.

$y=\frac{7}{8}x+\frac{1}{4}$

b. The graph of a linear relation has a slope -5 and a y-intercept of 3. Write the equation for the relation.

$m=-5$ and $b=3$ in $y=mx+b$.

$y=-5x+3$

⑥ 4. a. Two points of the graph of a linear relation are $(-8, -4)$ and $(0, 8)$. Write the equation for the relation.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{8-(-4)}{0-(-8)}$$

$$m = \frac{12}{8}$$

$$m = \frac{3}{2}$$

The point $(0, 8)$ is the y-intercept, so $b = 8$.

$$y = \frac{3}{2}x + 8$$

b. Two points of the graph of a linear relation are $(1, 3)$ and $(6, 5)$. Write the equation for the relation.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{5-3}{6-1}$$

$$= \frac{2}{5}$$

The point $(1, 3)$ is in the equation $y = mx + b$.

$$y = mx + b$$

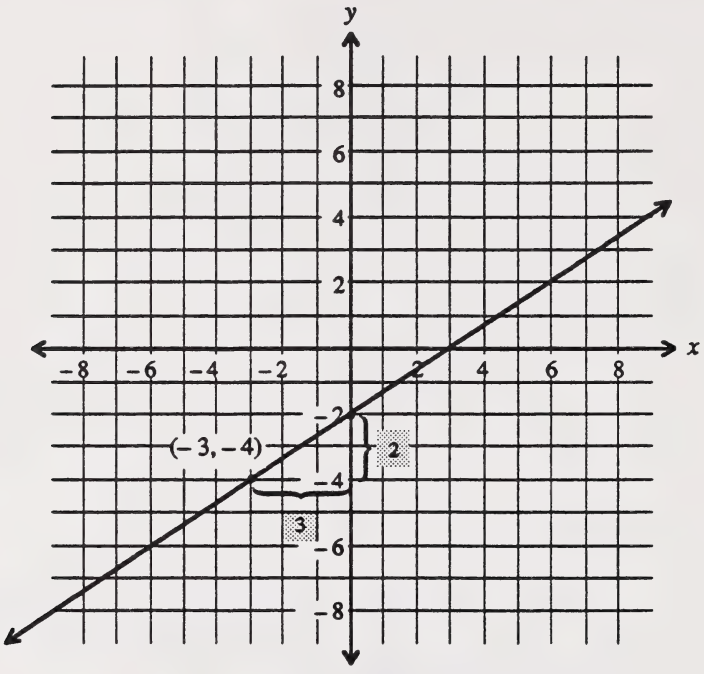
$$3 = \frac{2}{5}(1) + b$$

$$3 = \frac{2}{5} + b$$

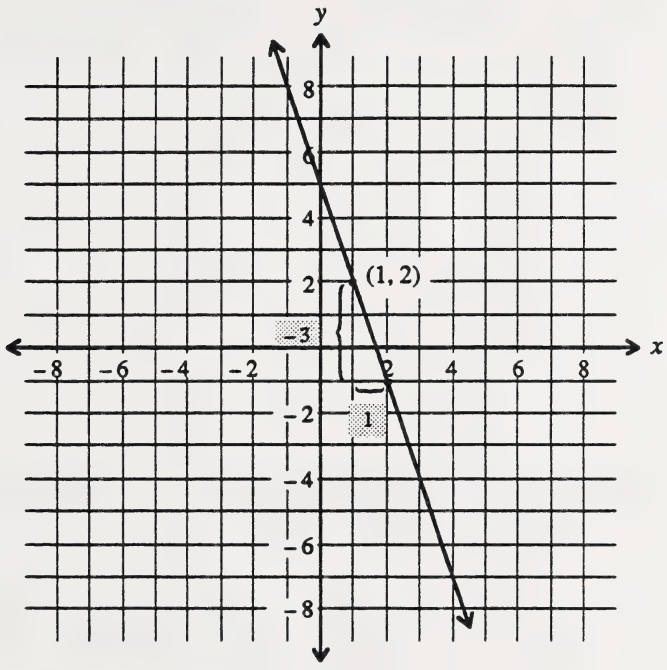
$$b = 2\tfrac{3}{5}$$

$$y = \tfrac{2}{5}x + 2\tfrac{3}{5}$$

④ 5. a. The graph of a linear relation has a slope of $\frac{2}{3}$ and contains the point $(-3, -4)$. Draw the graph of the relation.



④ b. The graph of a linear relation has a slope of -3 and contains the point $(1, 2)$. Draw the graph of the relation.



⑨ 6. a. The graph of a linear relation has a slope $\frac{8}{7}$ and contains the point $(49, 56)$. Write the equation of the linear relation.

$$y = mx + b$$
$$\uparrow \quad \uparrow$$
$$56 \quad 49$$

$$56 = \frac{8}{7}(49) + b$$
$$56 = 56 + b$$
$$0 = b$$
$$y = \frac{8}{7}x + 0 \text{ or } y = \frac{8}{7}x.$$

b. The graph of a linear relation has a slope of $\frac{5}{9}$ and contains the point (32, 0). Write the equation of the linear relation.

$$\begin{array}{c} \frac{5}{9} \\ \downarrow \\ y = mx + b \\ \uparrow \quad \uparrow \\ 0 \quad 32 \end{array}$$

$$0 = \frac{5}{9}(32) + b$$

$$0 = \frac{160}{9} + b$$

$$b = -\frac{160}{9}$$

$$b = -17\frac{7}{9}$$

$$y = \frac{5}{9}x - 17\frac{7}{9}$$

c. The graph of a linear relation has a slope of -5 and contains the point $(-5, 16)$. Write the equation of the linear relation.

$$\begin{array}{c} -5 \\ \downarrow \\ y = mx + b \\ \uparrow \quad \uparrow \\ 16 \quad -5 \end{array}$$

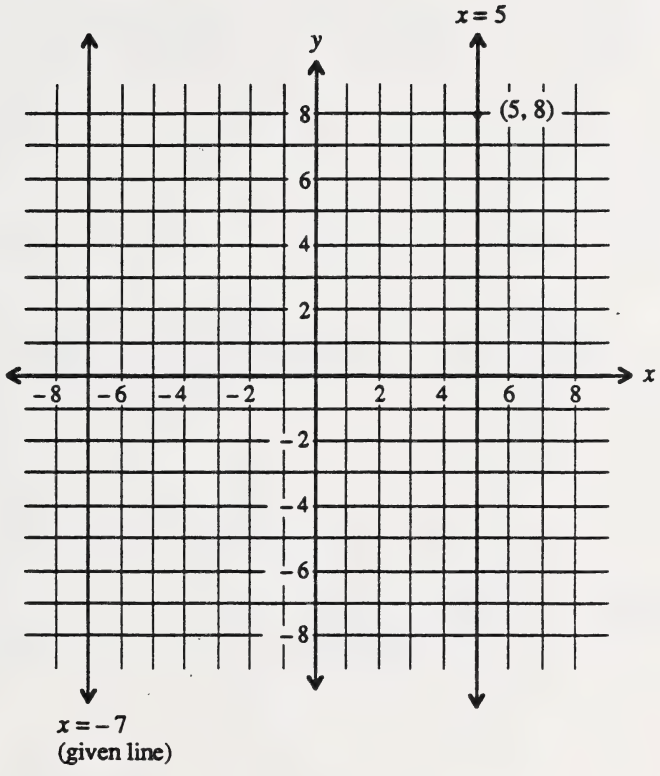
$$16 = -5(-5) + b$$

$$16 = 25 + b$$

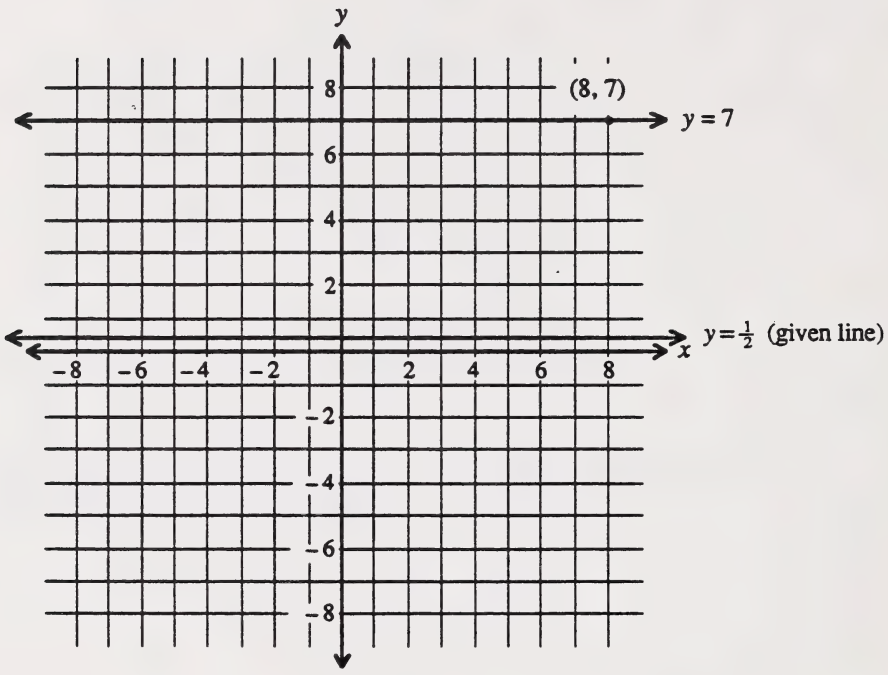
$$-9 = b$$

$$y = -5x - 9$$

④ 7. a. Draw the graph of the linear relation which contains the point (5, 8) and is parallel to $x = -7$.



④ b. Draw the graph of the linear relation which contains the point (8, 7) and is parallel to $y = \frac{1}{2}$.



(6) 8. a. Write the equation for the linear relation whose graph contains $(-1, 5)$ and is perpendicular to $y = -2$.

The line $y = -2$ is horizontal. The perpendicular line through the given point is a vertical line through $(-1, 5)$. The equation of such a line is $x = -1$.

b. Write the equation for the linear relation whose graph contains $(2, -7)$ and is perpendicular to $x = 5$.

The line $x = 5$ is vertical. The perpendicular line through the given point is a horizontal line through $(2, -7)$. The equation of such a line is $y = -7$.

(3) 9. Which of the following is perpendicular to $x - y = 0$?

A. $y - x = 0$
B. $x - y = 1$
C. $x + y = 0$
D. $x - y = -1$

C. The slope of the given line is 1 and the slope of the line for c is -1. The product of these slopes is $(1)(-1) = -1$.

(3) 10. Which of the following is parallel to $3x - 5y = 2$?

A. $y = \frac{3}{5}x - 2$

B. $y = -\frac{3}{5}x + 5$

C. $y = \frac{5}{3}x - 3$

D. $y = -\frac{5}{3}x + 7$

A. The slope of the given line is $\frac{3}{5}$. The slope for line a is $\frac{3}{5}$.

③ 11. Write the equation of the linear relation which has a graph as follows.

a. The graph is perpendicular to the graph of $y = 4x - 2$ and the graph contains the point (1, 3).

$$y = \underset{\uparrow}{3}\underset{\downarrow}{-\frac{1}{4}}x + \underset{\uparrow}{1}$$

$$m(4) = -1$$

$$m = -\frac{1}{4}$$

$$3 = -\frac{1}{4}(1) + b$$

$$3 = -\frac{1}{4} + b$$

$$b = 3\frac{1}{4}$$

$$y = -\frac{1}{4}x + 3\frac{1}{4}$$

③ b. The graph is parallel to $x - 5y = 2$ and contains the point (1, 3).

Find the slope of $x - 5y = 2$.

$$-5y = -x + 2$$

$$5y = x - 2$$

$$y = \frac{1}{5}x - \frac{2}{5}$$

$$m = \frac{1}{5}$$

$$y = \underset{\uparrow}{3}\underset{\downarrow}{\frac{1}{5}}x + \underset{\uparrow}{1}$$

$$3 = \frac{1}{5}(1) + b$$

$$3 = \frac{1}{5} + b$$

$$2\frac{4}{5} = b$$

$$y = \frac{1}{5}x + 2\frac{4}{5}$$

③ c. The graph is parallel to $y = \frac{1}{2}x + 5$ and contains the point $(-1, -4)$.

$\frac{1}{2}$
$y = mx + b$
$-4 \quad -1$

$$m = \frac{1}{2}$$
$$-4 = \frac{1}{2}(-1) + b$$
$$-4 = -\frac{1}{2} + b$$
$$-3\frac{1}{2} = b$$
$$y = \frac{1}{2}x - 3\frac{1}{2}$$

④ d. The graph is perpendicular to $3x - 2y = 5$ and contains the point $(-1, 5)$.

The slope of $3x - 2y = 5$
$$-2y = -3x + 5$$
$$2y = 3x - 5$$
$$y = \frac{3}{2}x - \frac{5}{2}$$
$$m = \frac{3}{2}$$

$-\frac{2}{3}$
$y = mx + b$
$5 \quad -1$

$$m\left(\frac{3}{2}\right) = -1$$
$$m = \frac{-1}{\frac{3}{2}}$$
$$m = \frac{-2}{3}$$
$$5 = \frac{-2}{3}(-1) + b$$
$$5 = \frac{2}{3} + b$$
$$4\frac{1}{3} = b$$
$$y = \frac{-2}{3}x + 4\frac{1}{3}$$

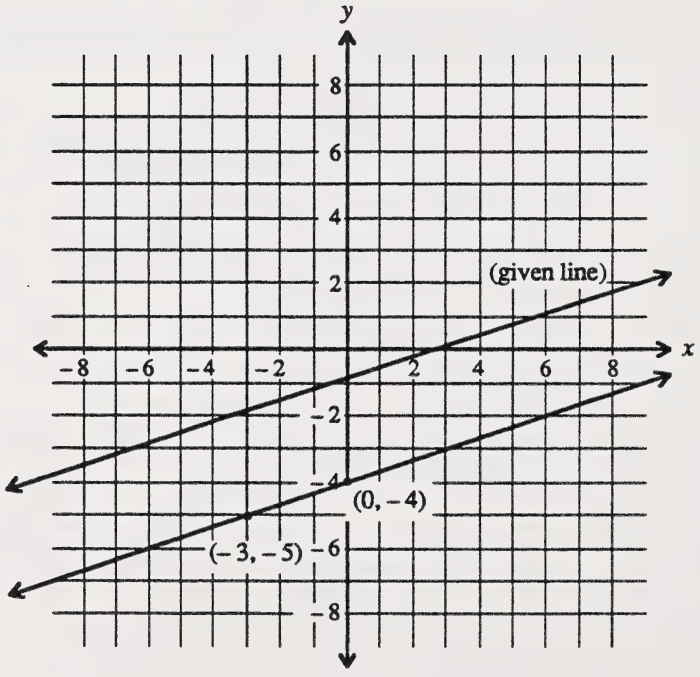
12. Write the equation and draw the graph of the linear relations which have the following conditions.

(6) a. One point of the graph is $(-3, -5)$ and the graph is parallel to the graph of $2x - 6y = 5$.

$$2x - 6y = 5$$
$$-6y = -2x + 5$$
$$6y = 2x - 5$$
$$y = \frac{1}{3}x - \frac{5}{6}$$
$$m = \frac{1}{3}$$

$$y = \underset{\uparrow}{\overset{\frac{1}{3}}{\downarrow}} mx + b$$
$$\underset{-5}{\uparrow} \quad \underset{-3}{\uparrow}$$

$$-5 = \frac{1}{3}(-3) + b$$
$$-5 = -1 + b$$
$$b = -4$$
$$y = \frac{1}{3}x - 4$$



⑥ b. One point of the graph is $(-2, 5)$ and the graph is perpendicular to the graph of $x - 5y = 3$.

$$x - 5y = 3$$
$$-5y = -x + 3$$
$$5y = x - 3$$
$$y = \frac{1}{5}x - \frac{3}{5}$$

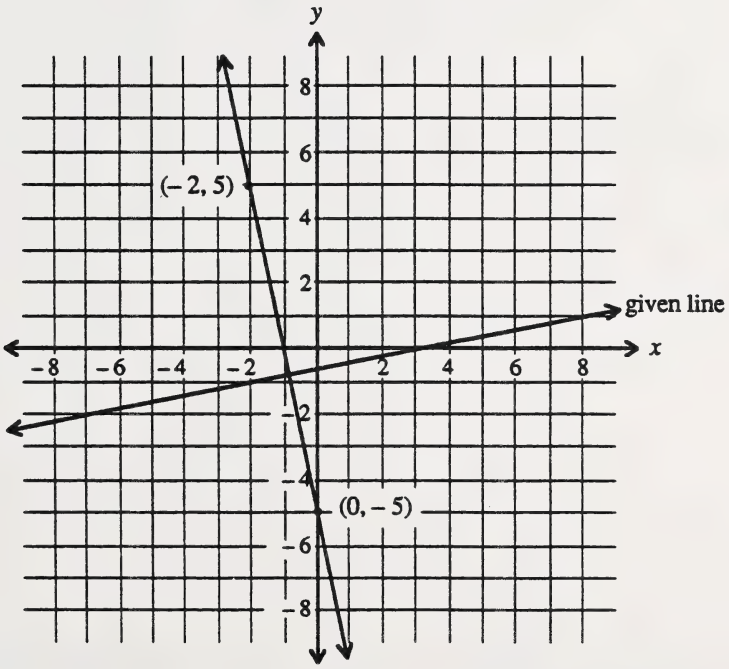
$$m = \frac{1}{5}$$

$$m\left(\frac{1}{5}\right) = -1$$

$$m = \frac{-1}{\frac{1}{5}}$$

$$m = -5$$

$$y = mx + b$$
$$5 = -5(-2) + b$$
$$5 = 10 + b$$
$$-5 = b$$
$$y = -5x - 5$$



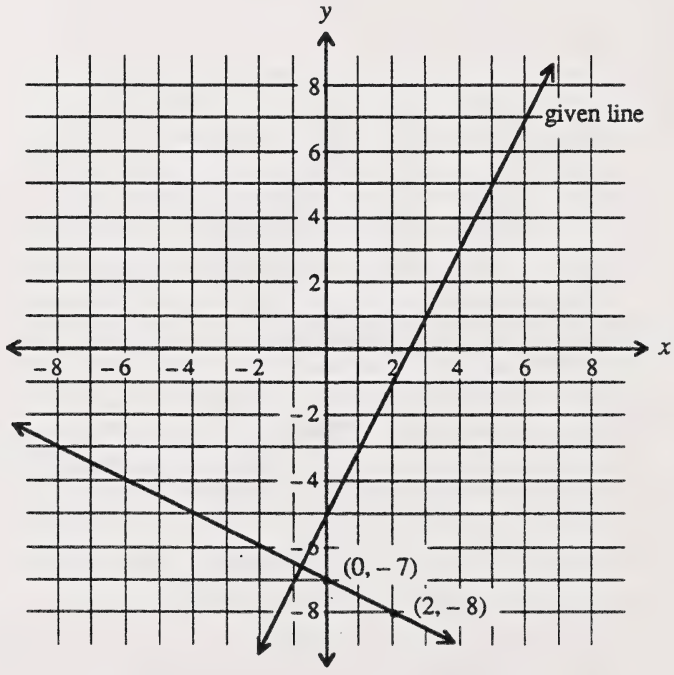
⑥ c. One point of the graph is $(0, -7)$ and the graph is perpendicular to the graph of $2x - y = 5$.

$$2x - y = 5$$
$$-y = -2x + 5$$
$$y = 2x - 5$$
$$m = 2$$

$$m(2) = -1$$
$$m = \frac{-1}{2}$$

$$y = mx + b$$
($m = -\frac{1}{2}$, $b = -7$)

$$-7 = -\frac{1}{2}(0) + b$$
$$-7 = 0 + b$$
$$-7 = b$$
$$y = -\frac{1}{2}x - 7$$



Topic 1

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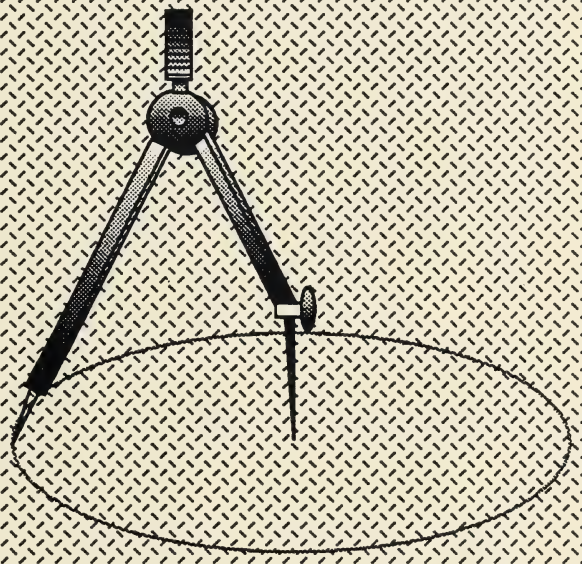
MATHEMATICS 23

LEARNING FACILITATOR'S MANUAL

UNIT 5

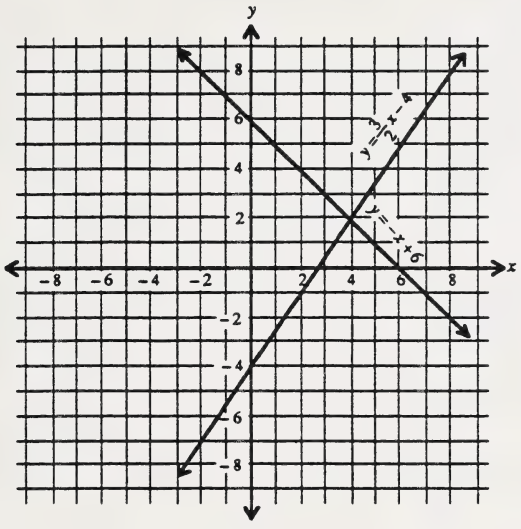


Distance Learning

Alberta EDUCATION

Topic 1: Solving Systems of Equations

(1) 1. a. Examine the graph below to determine the solution to the systems of equations.



The solution is (**4, 2**).

(2) b. Check if the above solution satisfies both equations. Show your work.

Check: $x = 4, y = 2$

LS	RS
y	$\frac{3}{2}x-4$
2	$\frac{3}{2}(4)-4$
	$6-4$
	2

LS = RS

LS	RS
y	$-x+6$
2	$-(4)+6$
	2

LS = RS

(5) 2. Determine the solution for each system of equations by graphing both equations on the grid provided. Complete the table of values before plotting the graphs. Check your solution.

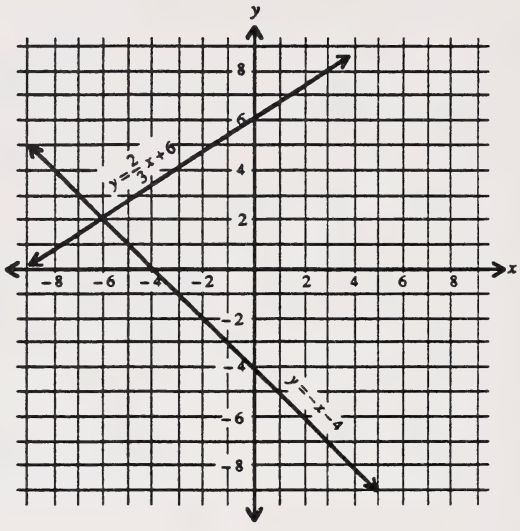
a. $y=\frac{2}{3}x+6;\ y=-x-4$

$y=\frac{2}{3}x+6$

x	0	-9
y	6	0

$y=-x-4$

x	0	-4
y	-4	0



The solution is $(-6, 2)$.

Check: $x=-6, y=2$

LS	RS
y	$\frac{2}{3}x+6$
2	$\frac{2}{3}(-6)+6$
	$-4+6$
	2

LS = RS

LS	RS
y	$-x-4$
2	$-(-6)-4$
	$6-4$
	2

LS = RS

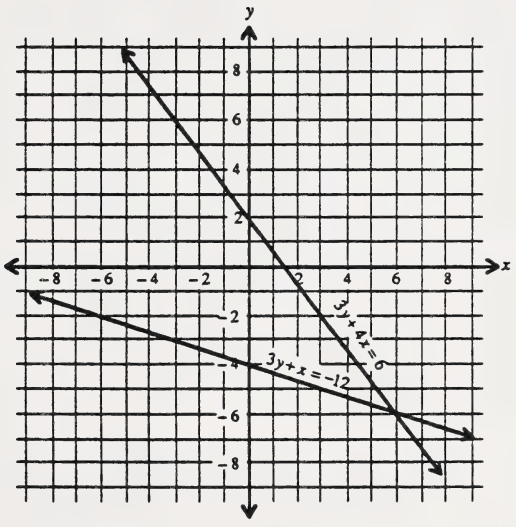
⑤ b. $3y + 4x = 6;\ 3y + x = -12$

$3y + 4x = 6$

x	0	1.5	6
y	2	0	−6

$3y + x = -12$

x	0	−6	
y	−4	−2	



The solution is $(6, -6)$.

Check: $x = 6, y = -6$

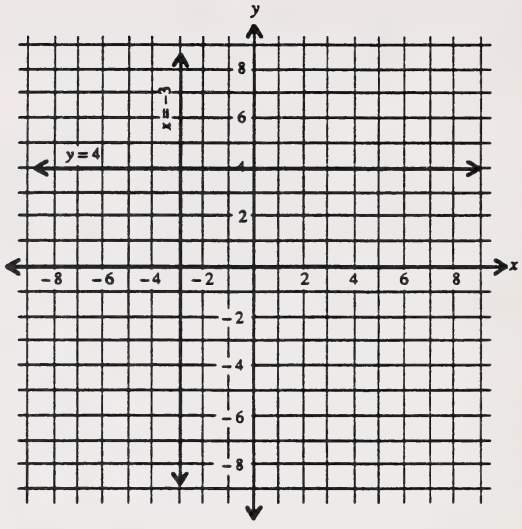
LS	RS
$3y + 4x$	6
$3(-6) + 4(6)$	
$-18 + 24$	
6	

LS = RS

LS	RS
$3y + x$	−12
$3(-6) + 6$	
$-18 + 6$	
-12	

LS = RS

(2) c. $x = -3;\ y = 4$



The solution is $(-3, 4)$.

(2) 3. Graph the following pairs of equations on the grid provided. Determine whether there are no solutions, one solution, or an infinite number of solutions.

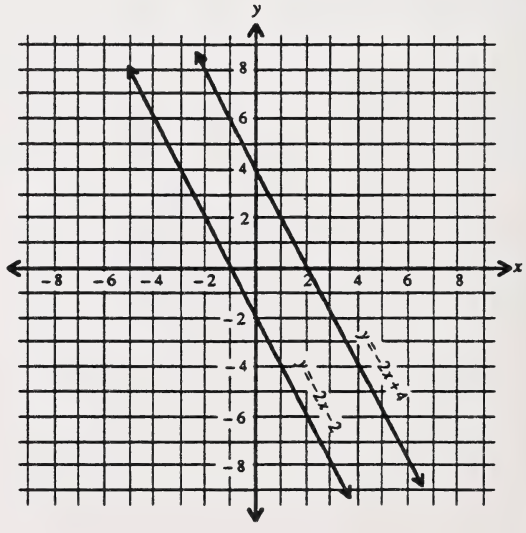
a. $y = -2x + 4;\ 2x + y = -2$

For $y = -2x + 4$,

x	-2	0	2
y	8	4	0

For $2x + y = -2$,

x	-2	0	2
y	2	-2	-6



There are no solutions. (These lines are parallel and they will never cross.)

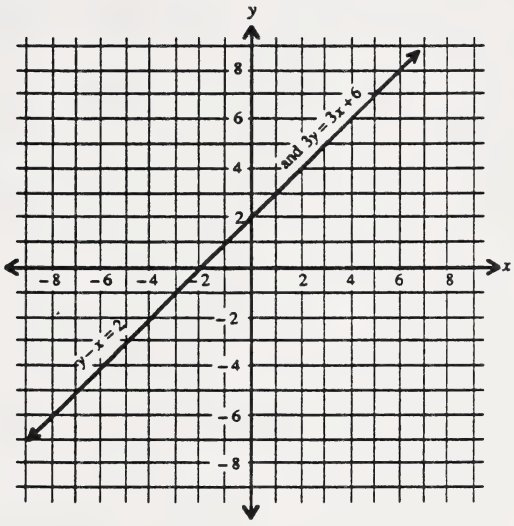
② b. $y - x = 2;\ 3y = 3x + 6$

For $y - x = 2$,

x	-2	0	2
y	0	2	4

For $3y = 3x + 6$,

x	-2	0	2
y	0	2	4



There is an infinite number of solutions. (These lines are exactly the same.)

(5) 4. Solve each system of equations by the elimination method. Then check your solutions for both equations.

a. $2x-4y=-22$
$x+2y=5$

$2x-4y=-22$ ①
$x+2y=5$ ②

$2\times$②: $2x+4y=10$ ③
$2x-4y=-22$ ①
③ + ①: $4x=-12$
$x=-3$

①: $2x-4y=-22$
$2(-3)-4y=-22$
$-6-4y=-22$
$-4y=-16$
$y=4$

Check: $x=-3, y=4$

LS	RS
$2x-4y$	-22
$2(-3)-4(4)$	
$-6-16$	
-22	

LS = RS

LS	RS
$x+2y$	5
$-3+2(4)$	
$-3+8$	
5	

LS = RS

The solution is $(-3, 4)$.

⑤ b. $3x-2y=17.5$
$2x+3y=-10$

$3x-2y=17.5$ ①
$2x+3y=-10$ ②

$3\times$ ①: $9x-6y=52.5$ ③
$2\times$ ②: $4x+6y=-20$ ④
③ + ④: $13x = 32.5$
$x = 2.5$

②: $2x+3y=-10$
$2(2.5)+3y=-10$
$5+3y=-10$
$3y=-15$
$y=-5$

Check: $x = 2.5, y = -5$

LS	RS
$3x-2y$	17.5
$3(2.5)-2(-5)$	
$7.5+10$	
17.5	

LS = RS

LS	RS
$2x+3y$	-10
$2(2.5)+3(-5)$	
$5-15$	
-10	

LS = RS

The solution is (2.5, – 5).

⑤ c. $3y = -x - 5$
$2x = y - 3$

$$x + 3y = -5 \quad ①$$
$$2x - y = -3 \quad ②$$

$3 \times ②$: $6x - 3y = -9$ ③
$x + 3y = -5$ ①
③ + ①: $7x = -14$
$x = -2$

②: $2x = y - 3$
$2(-2) = y - 3$
$2(-2) = y - 3$
$-4 = y - 3$
$y = -1$

Check: $x = -2, y = -1$

LS	RS
$3y$	$-x - 5$
$3(-1)$	$-(-2)$
-5	-3
$2 - 5$	
-3	

LS = RS

LS	RS
$2x$	$y - 3$
$2(-2)$	$-1 - 3$
-4	-4

LS = RS

The solution is $(-2, -1)$.

⑤ d. $x+y=4$

$\frac{1}{2}x+\frac{1}{3}y=3$

$x+y=4$ ①

$\frac{1}{2}x+\frac{1}{3}y=3$ ②

$6\times$ ②: $3x+2y=18$ ③

$3\times$ ①: $3x+3y=12$ ④

③ − ④: $-1y=6$

$y=-6$

①: $x+y=4$

$x+(-6)=4$

$x=4+6$

$x=10$

Check: $x=10, y=-6$

LS	RS
$x+y$	4
$10+(-6)$	
4	

LS = RS

LS	RS
$\frac{1}{2}x+\frac{1}{3}y$	3
$\frac{1}{2}(10)+\frac{1}{3}(-6)$	
$5-2$	
3	

LS = RS

The solution is $(10, -6)$.

⑤ e. $0.2x-0.3y=7$
$0.5x+0.7y=3$

$0.2x-0.3y=7$ ①
$0.5x+0.7y=3$ ②

$5\times$①: $1x-1.5y=35$ ③
$2\times$②: $1x+1.4y=6$ ④
③ − ④: $-2.9y=29$
$y=-10$

①: $0.2x-0.3y=7$
$0.2x-0.3(-10)=7$
$0.2x+3=7$
$0.2x=4$
$x=20$

Check: $x=20, y=-10$

LS	RS
$0.2x-0.3y$	7
$0.2(20)-0.3(-10)$	
$4+3$	
7	

LS = RS

LS	RS
$0.5x+0.7y$	3
$0.5(20)+0.7(-10)$	
$10-7$	
3	

LS = RS

The solution is (20, – 10).

⑤ f. $4x = 8y - 8$
$6y = 3 - 3x$

$$4x - 8y = -8 \quad ①$$
$$3x + 6y = 3 \quad ②$$

$3 \times ①$: $12x - 24y = -24$ ③
$4 \times ②$: $12x + 24y = 12$ ④
③+④: $24x = -12$

$$x = \frac{-12}{24}$$
$$x = -\frac{1}{2}$$

②: $3x + 6y = 3$
$3(-\frac{1}{2}) + 6y = 3$
$-\frac{3}{2} + 6y = 3$
$6y = 4.5$
$y = \frac{4.5}{6}$
$y = \frac{45}{60}$
$y = \frac{3}{4}$

Check: $x = -\frac{1}{2}$, $y = \frac{3}{4}$

LS	RS
$4x$	$8y - 8$
$4\left(-\frac{1}{2}\right)$	$8\left(\frac{3}{4}\right) - 8$
-2	$6 - 8$
	-2

LS = RS

LS	RS
$6y$	$3 - 3x$
$6\left(\frac{3}{4}\right)$	$3 - 3\left(-\frac{1}{2}\right)$
$4\frac{1}{2}$	$3 + \frac{3}{2}$
	$4\frac{1}{2}$

LS = RS

The solution is $\left(-\frac{1}{2}, \frac{3}{4}\right)$.

5. Complete the following statements, then discuss the solution set for each system.

(2) a. Two linear equations are each changed to the form $y=mx+b$. If the slopes are the same, but the y-intercepts are different, then the lines represented by these equations are parallel. Discuss the solution of this system.

There is no solution.

(2) b. Two linear equations are changed to the form $y=mx+b$. If the slopes are the same and the y-intercepts are the same, then the lines represented by these equations are the same. Discuss the solution of this system.

There is an infinite number of solutions.

(2) c. Two linear equations are each changed to the form $y=mx+b$. If the slopes are different, then the lines represented by these equations intersect. Discuss the solution of this system.

There is one solution.

For each problem, determine a system of two equations in two variables. Solve the system of equations algebraically; then check your answers in the original problem. Use a final statement to summarize what was determined.

(5) 6. The sum of two numbers in 36. Four times the smaller number exceeds the larger by 39. Determine the numbers.

Let x be the smaller number.
Let y be the larger number.

$$x + y = 36$$
$$4x - y = 39$$

$$x + y = 36 \quad ①$$
$$4x - y = 39 \quad ②$$

① + ②: $5x = 75$

$$x = 15$$

①: $x + y = 36$

$$15 + y = 36$$
$$y = 21$$

Check: $x = 15, y = 21$

LS	RS
$x+y$	36
$15+21$	
36	

LS = RS

LS	RS
$4x-y$	39
$60-21$	
39	

LS = RS

The smaller number is 15 and the larger number is 21.

(5) 7. An office cafeteria charges \$12.90 for four sandwiches and three drinks. Three sandwiches and two drinks cost \$9.45. Determine the individual cost for each sandwich and each drink.

Let x = the cost of one sandwich.
Let y = the cost of one drink.

$$4x+3y=12.90 \ (1)$$
$$3x+2y=9.45 \ (2)$$

$2\times(1)$: $8x+6y=25.80$ (4)
$3\times(2)$: $9x+6y=28.35$ (3)
(4) − (3): $-1x=-2.55$
(2): $x=2.55$

$$3x+2y=9.45$$
$$3(2.55)+2y=9.45$$
$$7.65+2y=9.45$$
$$2y=1.80$$
$$y=0.90$$

Check: $x=2.55$, $y=0.90$

LS	RS
$4x+3y$	12.90
$4(2.55)+3(0.90)$	
$10.20+2.70$	
12.90	

LS = RS

LS	RS
$3x+2y$	9.45
$3(2.55)+2(0.90)$	
$7.65+1.80$	
9.45	

LS = RS

Each sandwich costs \$2.55 and each drink costs \$0.90.

8. A rectangular acreage has a perimeter of 420 m and the length is 150 m less than three times the width.

(5) a. Determine the length and width of the acreage. Remember that $P=2l+2w$.

Let x = width.
Let y = length.

$$2x + 2y = 420 \quad (1)$$
$$3x - y = 150 \quad (2)$$

$2 \times (2)$: $6x - 2y = 300$ (3)

$$2x + 2y = 420 \quad (1)$$

(3)+(1): $8x = 720$

$$x = 90$$

(1): $2x + 2y = 420$

$$2(90) + 2y = 420$$
$$180 + 2y = 420$$
$$2y = 240$$
$$y = 120$$

Check: $x = 9, y = 120$

LS	RS
$2x + 2y$	420
$2(90) + 2(120)$	
$180 + 240$	
420	

LS = RS

LS	RS
$3x - y$	150
$3(90) - 120$	
$270 - 120$	
150	

LS = RS

The width is 90 m and the length is 120 m.

(2) b. Determine the area of the acreage in square metres. The formula for the area of a rectangle is $A = lw$.

$A = lw$

$A = 120(90)$

$A = 10\,800$

The area is $10\,800\,m^2$.

(1) c. Use the conversion 1 hectare (ha) = $10\,000\,m^2$ to determine the number of hectares in the acreage.

$1\ ha = 10\,000\ m^2$

$$\frac{10\,800\,m^2}{10\,000\,m^2/ha} = 1.08\ ha$$

The number of hectares are 1.08 ha.

(1) d. Use the conversion 1 ha = 2.471 acres to determine the number of acres (to 2 decimal places) in the acreage.

1 ha = 2.471 acres

1.08 ha = 1.08 ha × 2.471 acres/ha

= 2.67 acres

The acreage has 2.67 acres.

⑤ 9. Three years ago a father was five times as old as his son. In seven years the father will be 2.5 times as old as the son. Determine the present ages of the father and son.

Let x = the present age of the father.
Let y = the present age of the son.

(Three years ago) $x-3=5(y-3)$ ①
(In seven years) $x+7=2.5(y+7)$ ②

$$\begin{aligned} x\ -5y &= -12 && \text{③ from ①} \\ x-2.5y &= 10.5 && \text{④ from ②} \\ \text{③}-\text{④}:\ -2.5y &= -22.5 \\ y &= 9 \end{aligned}$$

$$\begin{aligned} \text{③}:\ x-5y &= -12 \\ x-5(9) &= -12 \\ x &= -12+45 \\ x &= 33 \end{aligned}$$

Check: $x = 33, y = 9$

LS	RS
$x-3$	$5(y-3)$
$33-3$	$5(9-3)$
30	$5(6)$
	30

LS = RS

LS	RS
$x+7$	$2.5(y+7)$
$33+7$	$2.5(9+7)$
40	$2.5(16)$
	40

LS = RS

The father's present age is 33 and the son's present age is 9.

(5) 10. The deposit is \$0.35 for large plastic pop containers and \$0.20 for small plastic pop containers. The number of large containers is five more than twice the number of small containers. The total money collected is \$8.95 when the containers are returned for their deposit refund. Determine the number of containers of each size.

Let x = number of large containers.
Let y = number of small containers.

$$0.35x + 0.20y = 8.95 \quad (1)$$
$$x = 2y + 5$$
$$x - 2y = 5 \quad (2)$$
$10 \times (1)$: $$3.5x + 2y = 89.5 \quad (3)$$
$(2) + (3)$: $$4.5x = 94.5$$
$$x = 21$$

(2): $$x - 2y = 5$$
$$21 - 2y = 5$$
$$21 - 5 = 2y$$
$$16 = 2y$$
$$8 = y$$

Check: $x = 21, y = 8$

LS	RS
$0.35x + 0.20y$	8.95
$0.35(21) + 0.20(8)$	
$7.35 + 1.60$	
8.95	

LS = RS

LS	RS
x	$2y + 5$
21	$2(8) + 5$
	$16 + 5$
	21

LS = RS

There are 21 large containers and 8 small containers.

(5) 11. A turkey pen is four times as long as it is wide. A total of 660 m of wire fence is required to enclose the pen.

a. Determine the length and width of the turkey pen.

Let x = width.
Let y = length.

$$2x + 2y = 660 \quad (1)$$
$$y = 4x$$
$$4x - y = 0 \quad (2)$$

$2 \times (2)$: $8x - 2y = 0 \quad (3)$
$2x + 2y = 660 \quad (1)$

$(1) + (3)$: $10x = 660$
$x = 66$

(2): $y = 4(x)$
$= 4(66)$
$= 264.$

Check: $x = 66, y = 264$

LS	RS
$2x + 2y$	660
$2(66) + 2(264)$	
$132 + 528$	
660	

LS = RS

LS	RS
y	$4x$
264	$4(66)$
	264

LS = RS

The length is 264 m and the width is 66 m.

(1)

b. Fence posts are required every 3 m to enclose the pen.
Determine the number of fence posts required.

660 m ÷ 3 m/post = 220 posts

The number of posts required are 220.

(2)

c. If fence posts cost $1.15 each and the wire fence costs $1.08/m, determine the total cost for the posts and wire fencing.

$$220 \times \$1.15 = \$253.00$$
$$660 \times \$1.08 = \underline{\$712.80}$$
$$\text{Total cost} = \$965.80$$

The total cost of the fence will be $965.80.

(5) 12. The stadium where the Red Deer Mustangs play their games has 15 000 seats.

a. The club plans to sell tickets for red section seats for \$10 and blue section seats for \$8. How many seats are in each section if the club plans to collect \$127 528 from ticket sales for each game?

Let x be the number of red section seats and
y be the number of blue section seats.

The income from red section seats will be $\$10x$ and
the income from blue section seats will be $\$8y$.

From the question: $x + y = 15\,000$ (1)
$10x + 8y = 127\,528$ (2)

Rearrange (1): $y = 15\,000 - x$ (3)

Substitute (3) into (2): $10x + 8(15\,000 - x) = 127\,528$
$10x + 120\,000 - 8x = 127\,528$
$2x + 120\,000 = 127\,528$
$2x = 7528$
$x = 3764$

(1): $x + y = 15\,000$
$3764 + y = 15\,000$
$y = 11\,236$

Check: $x = 3764$, $y = 11\,236$

LS	RS
$x + y$	15 000
3764 + 11 236	
15 000	
LS = RS	

LS	RS
$10x + 8y$	127 528
10(3764) + 8(11 236)	
37 640 + 89 888	
127 528	
LS = RS	

There are 3764 red seats and 11 236 blue seats.

(3) b. If 2500 seats are added to the blue section, how much income can the club expect from ticket sales?

$2500 \times \$8 = \$20\,000$

$20 000 will be raised from the new seats.

$$\begin{array}{r} \$127\,528 \\ +\ 20\,000 \\ \hline \$147\,528 \end{array}$$

The total income from the tickets will be $147 528.

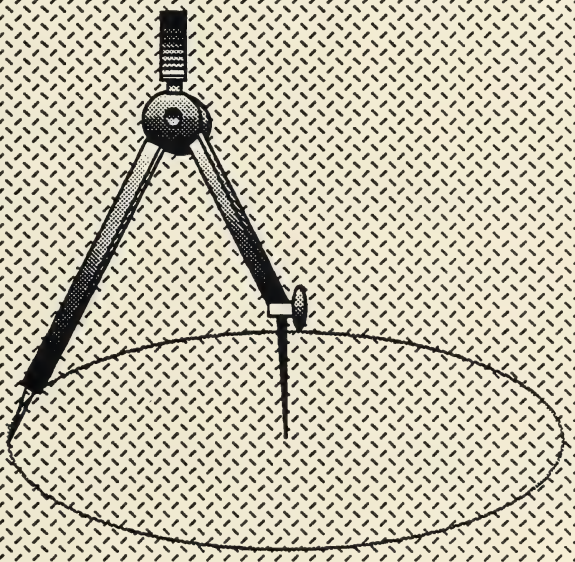
Topic 1

_____ marks

MATHEMATICS 23

LEARNING FACILITATOR'S MANUAL

UNIT 6



Distance Learning

Alberta EDUCATION

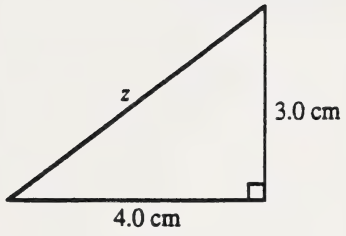
Topic 1: Perimeter

③ 1. What is the meaning of the perimeter of a figure?

Perimeter is the distance around a figure.

③ 2. What is the perimeter of this right triangle?

$$z^2 = (3.0\text{ cm})^2 + (4.0\text{ cm})^2$$
$$= 9.00\text{ cm}^2 + 16.00\text{ cm}^2$$
$$= 25.00\text{ cm}^2$$
$$z = 5.0\text{ cm}$$
$$P = 3.0\text{ cm} + 4.0\text{ cm} + 5.0\text{ cm}$$
$$= 12.0\text{ cm}$$



The perimeter of the triangle is 12.0 cm.

③ 3. What is the perimeter of a regular hexagon in which each side measures 2.125 m?

$$P = ns$$
$$= 6(2.125\text{ m})$$
$$= 12.75\text{ m}$$
$$\doteq 12.8\text{ m}$$

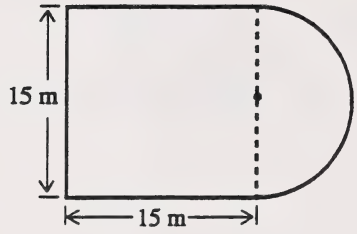
The perimeter of the hexagon is 12.8 m.

③ 4. Suppose that it was recorded that a bicycle wheel rotates ten times in a distance of 18.78 m. The wheel has a diameter of 60.0 cm. Based on this information, what would you conclude about the value of π?

$$C = \frac{18.78\text{ m}}{10}$$
$$= 1.878\text{ m}$$
$$= 187.8\text{ cm}$$
$$C = \pi d$$
$$\pi = \frac{C}{d}$$
$$= \frac{187.8\text{ cm}}{60.0\text{ cm}}$$
$$= 3.13$$

Based on this information, $\pi = 3.13$.

(3) 5. What is the perimeter of this figure? (The curved part forms a semicircle.)



The curved part has a length of $\frac{1}{2}C = \frac{1}{2}(\pi d)$

$= \pi r$

$= (\pi)(7.5 \text{ m})$

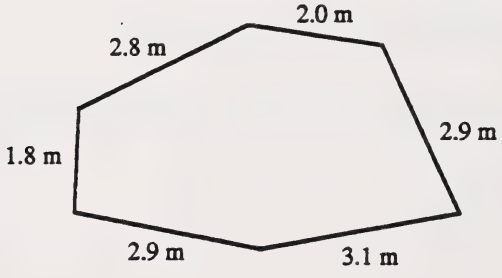
$\doteq 23.56 \text{ m}$

$P = 15 \text{ m} + 15 \text{ m} + 15 \text{ m} + 23.56 \text{ m}$

$\doteq 68.56 \text{ m}$

The perimeter of the figure is 68.56 m.

(3) 6. What is the perimeter of this figure?



$P = 2.8 \text{ m} + 2.0 \text{ m} + 2.9 \text{ m} + 3.1 \text{ m} + 2.9 \text{ m} + 1.8 \text{ m}$

$= 15.5 \text{ m}$

Topic 1 The perimeter of the figure is 15.5 m.

_____ marks

Topic 2: Area

③ 1. What is the meaning of the area of a figure?

The area of a figure is the number of square units enclosed by the figure.

③ 2. In a certain region of Alberta it has been found that 25 000 m^2 of pasture are needed to support one cow. How many cows can be supported by a square paddock having side lengths of 1.0 km?

$A = (1000\ m)(1000\ m)$

$= 1\ 000\ 000\ m^2$

The paddock can support $\frac{1\ 000\ 000\ m^2}{25\ 000\ m^2/cow} = 40$ cows.

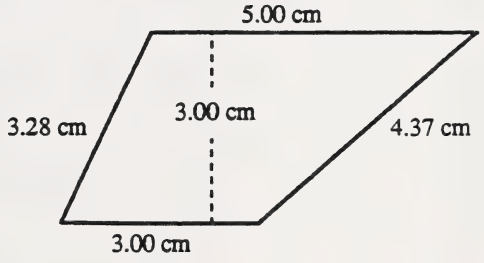
③ 3. What is the area of the trapezoid?

$A = \frac{1}{2}h(a+b)$

$= \frac{1}{2}(3.00\ cm)(5.00\ cm + 3.00\ cm)$

$= 12.0\ cm^2$

The area of the trapezoid is 12.0 cm^2.



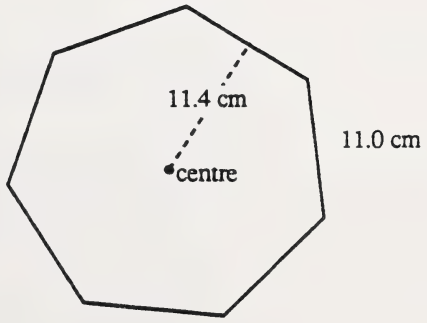
③ 4. What is the area of the regular heptagon? Do not round the answer.

$A = \frac{1}{2}ans$

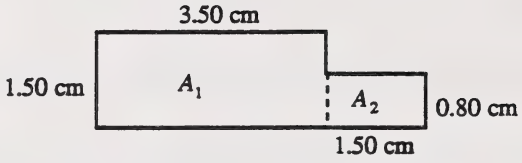
$= \frac{1}{2}(11.4\ cm)(7)(11.0\ cm)$

$= 438.9\ cm^2$

The area of the heptagon is 438.9 cm^2.



(3) 5. What is the area of the hexagon?



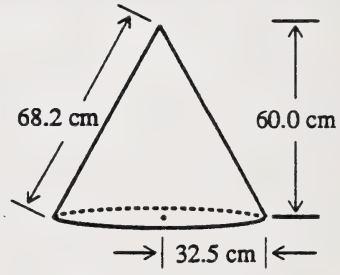
$$A = A_1 + A_2$$
$$= l_1 w_1 + l_2 w_2$$
$$= (3.50\text{ cm})(1.50\text{ cm}) + (1.50\text{ cm})(0.80\text{ cm})$$
$$= 5.25\text{ cm}^2 + 1.20\text{ cm}^2$$
$$= 6.45\text{ cm}^2$$

The area of the figure is 6.45 cm^2.

(3) 6. What is the surface area of the cone? Round the answer to the nearest whole square centimetre.

$$A = \pi r^2 + \pi rs$$
$$= (\pi)(32.5\text{ cm})^2 + (\pi)(32.5\text{ cm})(68.2\text{ cm})$$
$$\doteq 3318\text{ cm}^2 + 6963\text{ cm}^2$$
$$\doteq 10\,281\text{ cm}^2$$

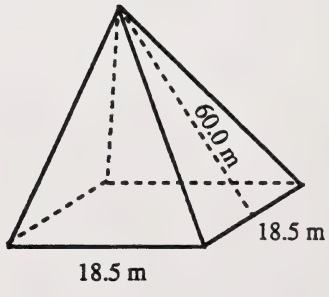
The surface area of the cone is $10\,281\text{ cm}^2$.



(3) 7. Determine the surface area of the pyramid. Do not round the answer.

$$A = 2bs + b^2$$
$$= 2(18.5\text{ m})(60.0\text{ m}) + (18.5\text{ m})^2$$
$$= 2220\text{ m}^2 + 342.25\text{ m}^2$$
$$= 2562.25\text{ m}^2$$

The surface area of the pyramid is 2562.25 m^2.



(3) 8. What is the surface area of a cannon ball having a diameter of 10.0 cm? Round the answer to the nearest whole square centimetre.

$$A = 4\pi r^2$$
$$= 4(\pi)(5.0 \text{ cm})^2$$
$$\doteq 314 \text{ cm}^2$$

The surface area of the cannon ball is 314 cm^2.

(3) 9. What is the surface area of a regular dodecahedron (12-sided figure) in which each face has an area of 13.25 cm^2?

$$A = A_f n$$
$$= (13.25 \text{ cm}^2)12$$
$$= 159 \text{ cm}^2$$

The surface area of the dodecahedron is 159 cm^2.

(3) 10. What is the side length of a square garden having an area of 625 m^2?

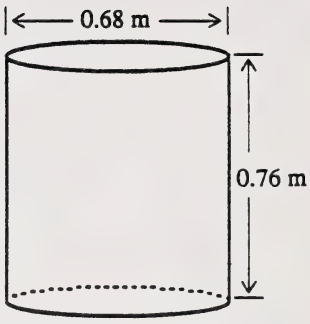
$$A = s^2$$
$$s = \sqrt{A}$$
$$= \sqrt{625 \text{ m}^2}$$
$$= 25 \text{ m}$$

The side length of the garden is 25 m.

(2) 11. What figure encloses the greatest area for a given perimeter? _____

The circle encloses the greatest area.

(3) 12. What is the surface area of the cylinder? Round the answer to two decimal places.



$$A = 2\pi rh + 2\pi r^2$$
$$= 2(\pi)(0.34\text{ m})(0.76\text{ m}) + 2(\pi)(0.34\text{ m})^2$$
$$\doteq 1.62\text{ m}^2 + 0.73\text{ m}^2$$
$$\doteq 2.35\text{ m}^2$$

The surface area of the cylinder is 2.35 m^2.

Topic 2

_____ marks

Topic 3: Volume

(3) 1. What is the meaning of the volume of a solid?

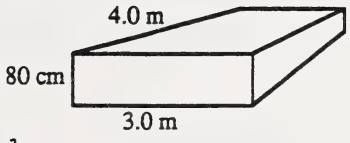
The volume is the number of cubic units in a solid.

(3) 2. What is the volume of the rectangular prism?

$V = lwh$

$= (4.0\text{ m})(3.0\text{ m})(0.80\text{ m})$

$= 9.6\text{ m}^3$



The volume of the rectangular prism is 9.6 m^3.

(3) 3. Suppose a cube has a volume of $42\ 875\text{ cm}^3$. Give the dimensions of this cube.

$V = s^3$

$s = \sqrt[3]{V}$

$= \sqrt[3]{42\ 875\text{ cm}^3}$

$= 35\text{ cm}$

The cube measures 35 cm by 35 cm by 35 cm.

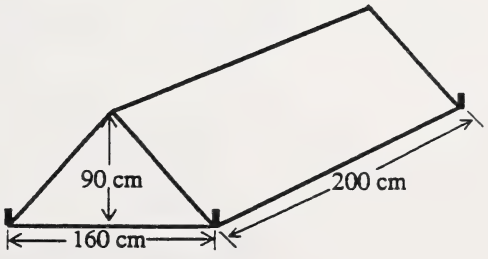
(3) 4. What is the volume of the pup tent?

$V = Bh$

$= \left[\frac{1}{2}(160\text{ cm})(90\text{ cm})\right](200\text{ cm})$

$= (7200\text{ cm}^2)(200\text{ cm})$

$= 1\ 440\ 000\text{ cm}^3 \text{ or } 1.44\text{ m}^3$

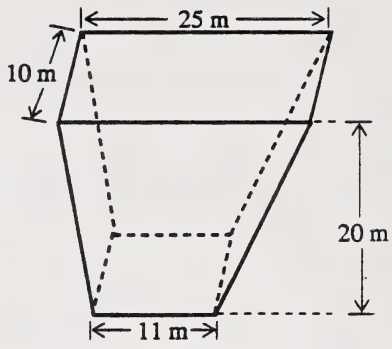


The volume of the pup tent is $1\ 440\ 000\text{ cm}^3$.

(3) 5. What is the volume of the prism? Each end forms a trapezoid.

$$B = \frac{1}{2}h(a+b)$$
$$= \frac{1}{2}(20\text{ m})(25\text{ m} + 11\text{ m})$$
$$= 360\text{ m}^2$$
$$V = Bh$$
$$= (360\text{ m}^2)(10\text{ m})$$
$$= 3600\text{ m}^3$$

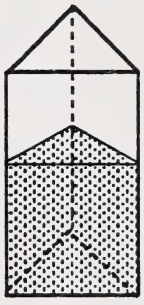
The volume of the prism is 3600 m^3.



(3) 6. The bottom of the prism-shaped container has an area of 100 cm^2. If the container is filled with 2 L of water, how high will the fluid surface be above the bottom?

$$V = Bh$$
$$h = \frac{V}{B}$$
$$= \frac{2\text{ L}}{100\text{ cm}^2}$$
$$= \frac{2000\text{ cm}^3}{100\text{ cm}^2}$$
$$= 20\text{ cm}$$

The fluid surface will be 20 cm above the bottom of the container.



(3) 7. A conservatory has a glass structure in the shape of a pyramid. The area of its base is 660 m^2, and its height is 24 m. What is the volume of the structure?

$$V = \frac{1}{3}Bh$$
$$= \frac{1}{3}(660\text{ m}^2)(24\text{ m})$$
$$= 5280\text{ m}^3$$

The volume of the pyramid is 5280 m^3.

(3) 8. What is the volume of a cylinder having a radius of 12.0 cm and a height of 20.0 cm? Round the answer to the nearest whole cubic centimetre.

$$V = \pi r^2 h$$
$$= (\pi)(12.0 \text{ cm})^2(20.0 \text{ cm})$$
$$\doteq 9048 \text{ cm}^3$$

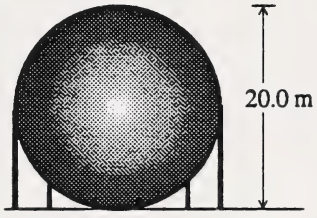
The volume of the cylinder is 9048 cm^3.

(3) 9. What is the volume of a cone having a height of 20.0 cm and a base with a radius of 12.0 cm? Round the answer to the nearest whole cubic centimetre.

$$V = \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3}(\pi)(12.0 \text{ cm})^2(20.0 \text{ cm})$$
$$\doteq 3016 \text{ cm}^3$$

The volume of the cone is 3016 cm^3.

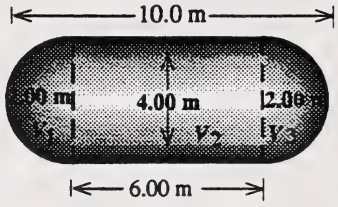
(3) 10. A spherical tank is supported so that its bottom surface rests at ground level. If the distance from the ground to the top of the tank is 20.0 m, what is the volume of the tank? Round the answer to the nearest whole cubic metre.



$$r = \frac{20.0 \text{ m}}{2}$$
$$= 10.0 \text{ m}$$
$$V = \frac{4}{3}\pi r^3$$
$$= \frac{4}{3}(\pi)(10.0)^3$$
$$\doteq 4189 \text{ m}^3$$

The volume of the tank is 4189 m^3.

(3) 11. What is the volume of this figure? The curved ends each form a hemisphere. Round the final answer to the nearest whole cubic metre.



$$
\begin{aligned}
V &= V_1 + V_2 + V_3 \\
&= (V_1 + V_3) + V_2 \\
&= \left(\frac{4}{3}\pi r^3\right) + \pi r^2 h \\
&= \frac{4}{3}(\pi)(2.00\text{ m})^3 + (\pi)(2.00\text{ m})^2(6.00\text{ m}) \\
&\doteq 33.5\text{ m}^3 + 75.4\text{ m}^3 \\
&\doteq 109\text{ m}^3
\end{aligned}
$$

The volume of the figure is 109 m^3.

(2) 12. For a fixed surface area, what figure has the greatest volume? _____

The sphere has the greatest volume.

Topic 3

_____ marks

Topic 4: Relationships for Geometric Figures

(3) 1. Determine the sum of the angles for a pentagon.

$$\begin{aligned}\text{Angle sum} &= 180°(n-2)\\ &= 180°(5-2)\\ &= 180°(3)\\ &= 540°\end{aligned}$$

The angle sum for a pentagon is 540°.

(3) 2. How many diagonals does a hexagon have?

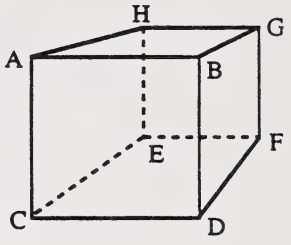
$$\begin{aligned}\text{The number of diagonals} &= \frac{1}{2}n(n-3)\\ &= \frac{1}{2}(6)(6-3)\\ &= 9\end{aligned}$$

A hexagon has 9 diagonals.

(4) 3. Determine the value of

V – E + F for the cube, where
V is the number of vertices,
E is the number of edges, and
F is the number of faces. In your work, list the edges, vertices, and faces.

Edges	Vertices	Faces
$\overline{AB}$ $\overline{DF}$ $\overline{AH}$ $\overline{EF}$ $\overline{AC}$ $\overline{EH}$ $\overline{BG}$ $\overline{FG}$ $\overline{BD}$ $\overline{GH}$ $\overline{CD}$ $\overline{CE}$	A, B, C, D, E, F, G, H	ABCD ACEH ABGH CDEF BDFG EFGH
Totals: 12	8	6



$V - E + F = 8 - 12 + 6$
$= 2$

For the cube, the value of V – E + F is 2.

(2) 4. How many diagonals does a pyramid have? _____

A pyramid has no diagonals.

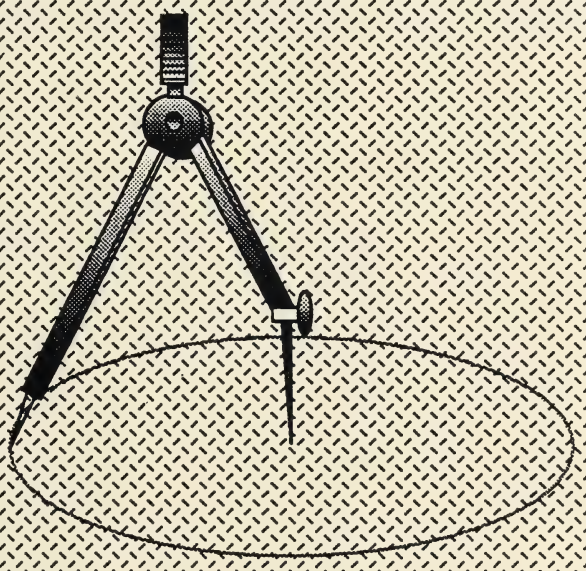
Topic 4

_____ marks

MATHEMATICS 23

LEARNING FACILITATOR'S MANUAL

UNIT 7

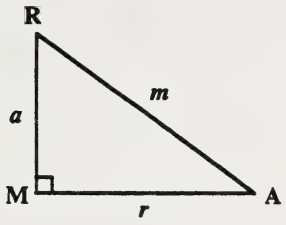


Distance Learning

Alberta EDUCATION

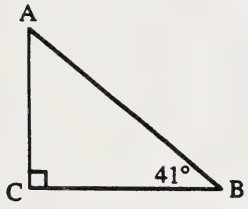
Topic 1: Properties of Triangles

(1) 1. Use the letters R, A, and M to label the given triangle.



Other arrangements are possible as long as a lower case letter is used opposite an upper case letter.

(1) 2. What is the measure of $\angle A$ in ΔABC?

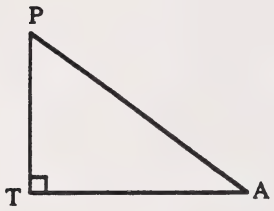


$\angle A = 180° - (90° + 41°)$

$\angle A = 180° - 131°$

$\angle A = 49°$

(2) 3. Use ΔPAT to answer the following.

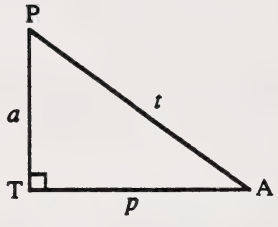


a. Identify the acute angles.

$\angle P$ and $\angle A$ are the acute angles.

b. Name the side opposite each acute angle.

The side opposite $\angle P$ is p.
The side oppsoite $\angle A$ is a.



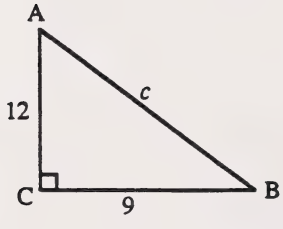
c. Name the side adjacent to each acute angle.

The side adjacent to $\angle P$ is a.
The side adjacent to $\angle A$ is p.

d. Identify the hypotenuse.

The hypotenuse is $\overline{AP}$ or t.

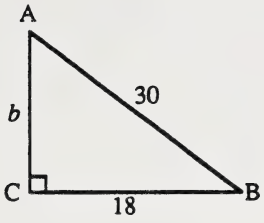
(2) 4. Find c in ΔABC.



$$c^2 = a^2 + b^2$$
$$c^2 = 9^2 + 12^2$$
$$c^2 = 81 + 144$$
$$c^2 = 225$$
$$\sqrt{c^2} = \sqrt{225}$$
$$c = 15$$

The value for c is 15.

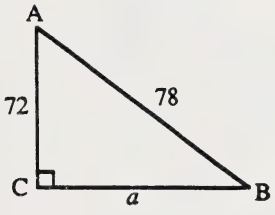
(2) 5. Find b in ΔABC.



$$b^2 = c^2 - a^2$$
$$b^2 = 30^2 - 18^2$$
$$b^2 = 576$$
$$\sqrt{b^2} = \sqrt{576}$$
$$b = 24$$

The value for b is 24.

(2) 6. Find a in ΔABC.



$$a^2 = c^2 - b^2$$
$$a^2 = 78^2 - 72^2$$
$$a^2 = 6084 - 5184$$
$$a^2 = 900$$
$$\sqrt{a^2} = \sqrt{900}$$
$$a = 30$$

The value for a is 30.

Topic 1

_____ marks

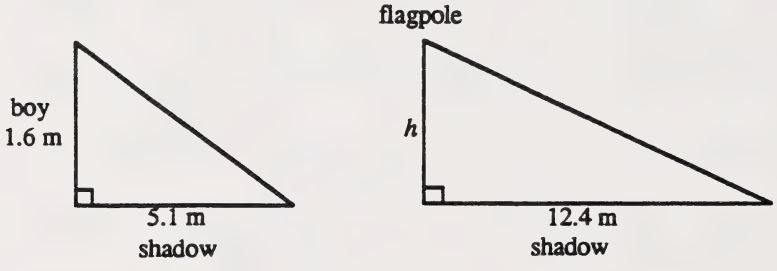
Topic 2: Similar Triangles

③ 1. $\Delta RAT \sim \Delta MOP$
List all the similarity relationships.

Angles:	Sides:
$\angle R \leftrightarrow \angle M$	$\overline{RA} \leftrightarrow \overline{MO}$
$\angle A \leftrightarrow \angle O$	$\overline{AT} \leftrightarrow \overline{OP}$
$\angle T \leftrightarrow \angle P$	$\overline{RT} \leftrightarrow \overline{MP}$

③ 2. A boy 1.6 m tall casts a shadow of 5.1 m. At the same time, a flagpole casts a shadow of 12.4 m.

What is the height of the flagpole? Round the answer to the nearest tenth.



$$\frac{1.6}{h} = \frac{5.1}{12.4}$$

$$5.1h = 1.6 \times 12.4$$

$$5.1h = 19.84$$

$$\frac{5.1h}{5.1} = \frac{19.84}{5.1}$$

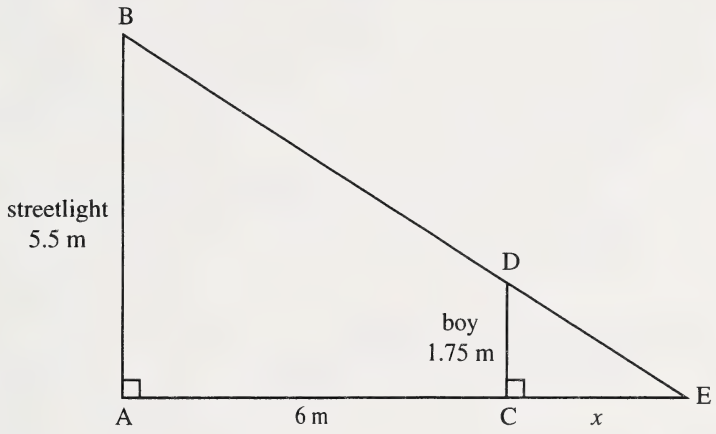
$$h \doteq 3.8902$$

$$h \doteq 3.9$$

The height of the flagpole is 3.9 m.

④ 3. A boy, 1.75 m tall, stands close to a streetlight that is 5.5 m tall. If the boy stands 6 m from the streetlight, calculate the length of the boy's shadow.

Let x be the length of the boy's shadow.



$$\frac{AB}{CD} = \frac{AE}{CE}$$

$$\frac{5.5}{1.75} = \frac{x+6}{x}$$

$$5.5x = 1.75x + 10.5$$

$$5.5x - 1.75x = 10.5$$

$$3.75x = 10.5$$

$$x = 2.8$$

Therefore, the length of the boy's shadow is 2.8 m.

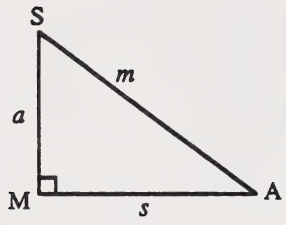
Topic 2

_____ marks

Topic 3: Developing the Trigonometric Ratios

(6) 1. a. Label a right triangle using S, A, and M.

This is one possible solution.



b. Identify the two acute angles.
The two acute angles are ∠S and ∠A.

c. Give the three primary trigonometric ratios for each angle.

For ∠A,

$\sin A = \frac{a}{m}$

$\cos A = \frac{s}{m}$

$\tan A = \frac{a}{s}$

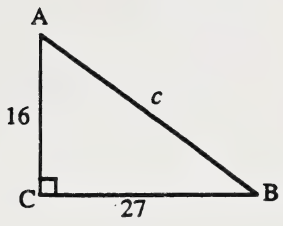
For ∠S,

$\sin S = \frac{s}{m}$

$\cos S = \frac{a}{m}$

$\tan S = \frac{s}{a}$

⑥ 2. Using ΔABC to answer the following.



a. Find c. Round the answer to the nearest whole number.

$$c^2 = a^2 + b^2$$
$$c^2 = 27^2 + 16^2$$
$$c^2 = 729 + 256$$
$$c^2 = 985$$
$$\sqrt{c^2} = \sqrt{985}$$
$$c \doteq 31.385$$
$$c \doteq 31$$

The value for c is approximately 31.

b. Determine the three primary trigonometric ratios for each acute angle. Round the answers to four decimal places.

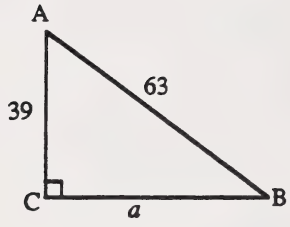
For $\angle A$,

$$\sin A = \frac{27}{31} \doteq 0.8710$$
$$\cos A = \frac{16}{31} \doteq 0.5161$$
$$\tan A = \frac{27}{16} = 1.6875$$

For $\angle B$,

$$\sin B = \frac{16}{31} \doteq 0.5161$$
$$\cos B = \frac{27}{31} \doteq 0.8710$$
$$\tan B = \frac{16}{27} \doteq 0.5926$$

⑥ 3. Use ΔABC to answer the following.



a. Find a, correct to one decimal place.

$$a^2 = c^2 - b^2$$
$$a^2 = 63^2 - 39^2$$
$$a^2 = 3969 - 1521$$
$$a^2 = 2448$$
$$\sqrt{a^2} = \sqrt{2448}$$
$$a \doteq 49.477$$
$$a \doteq 49.5$$

The value for a is approximately 49.5.

b. Find the three primary trigonometric ratios for each acute angle. Round the answers to four decimal places.

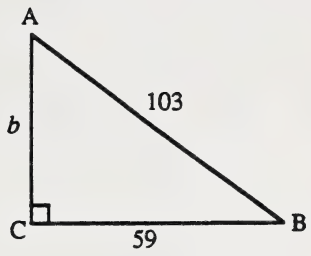
For $\angle A$,

$$\sin A = \frac{49.5}{63} \doteq 0.7857$$
$$\cos A = \frac{39}{63} \doteq 0.6190$$
$$\tan A = \frac{49.5}{39} \doteq 1.2692$$

For $\angle B$,

$$\sin B = \frac{39}{63} \doteq 0.6190$$
$$\cos B = \frac{49.5}{63} \doteq 0.7857$$
$$\tan B = \frac{39}{49.5} \doteq 0.7879$$

⑥ 4. Use ΔABC to answer the following.



a. Find b, correct to one decimal place.

$$b^2 = c^2 - a^2$$
$$b^2 = 103^2 - 59^2$$
$$b^2 = 10\,609 - 3481$$
$$b^2 = 7128$$
$$\sqrt{b^2} = \sqrt{7128}$$
$$b \doteq 84.427$$
$$b \doteq 84.4$$

The value for b is approximately 84.4.

b. Find the primary trigonometric ratios for each acute angle. Round the answers to four decimal places.

For $\angle A$,

$$\sin A = \frac{59}{103} \doteq 0.5728$$
$$\cos A = \frac{84.4}{103} \doteq 0.8194$$
$$\tan A = \frac{59}{84.4} \doteq 0.6991$$

For $\angle B$,

$$\sin B = \frac{84.4}{103} \doteq 0.8194$$
$$\cos B = \frac{59}{103} \doteq 0.5728$$
$$\tan B = \frac{84.4}{59} \doteq 1.4305$$

(6) 5. Find the measure of $\angle A$ in each of the following cases. Express each measure to the nearest degree.

a. $\tan A = 0.9906$

To the nearest degree, $\angle A = 45°$.

b. $\sin A = 0.3009$

To the nearest degree, $\angle A = 18°$.

c. $\cos A = 0.8346$

To the nearest degree, $\angle A = 33°$.

d. $\tan A = 3.0049$

To the nearest degree, $\angle A = 72°$.

e. $\cos A = 0.4366$

To the nearest degree, $\angle A = 64°$.

f. $\sin A = 0.9715$

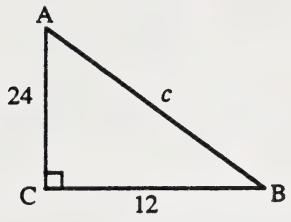
To the nearest degree, $\angle A = 76°$.

Topic 3

_____ marks

Topic 4: Solving Right Triangles

(8) 1. Use ΔABC to answer the following.



a. To the nearest whole number, find the measure of the unknown side.

Find c.

$$c^2 = a^2 + b^2$$
$$c^2 = 12^2 + 24^2$$
$$c^2 = 144 + 576$$
$$c^2 = 720$$
$$\sqrt{c^2} = \sqrt{720}$$
$$c \doteq 26.833$$
$$c \doteq 27$$

The measure of c is approximately 27.

b. Find the measure of each acute angle to the nearest degree. Use all three possibilities for each angle.

Find the measure of $\angle A$.

$$\sin A = \frac{12}{27}$$
$$\doteq 0.4444$$

To the nearest degree, $\angle A = 26°$.

$$\cos A = \frac{24}{27}$$
$$\doteq 0.8889$$

To the nearest degree, $\angle A = 27°$.

$$\tan A = \frac{12}{24}$$
$$= 0.5000$$

To the nearest degree, $\angle A = 27°$.

Each method results in almost the same measure for angle A. The difference is due to the rounding-off procedures.

Find the measure of $\angle B$.

$$\sin B = \frac{24}{27}$$
$$\doteq 0.8889$$

To the nearest degree, $\angle B = 63°$.

$$\cos B = \frac{12}{27}$$
$$\doteq 0.4444$$

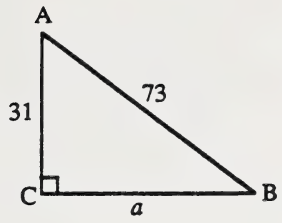
To the nearest degree, $\angle B = 64°$.

$$\tan B = \frac{24}{12}$$
$$= 2.000$$

To the nearest degree, $\angle B = 63°$.

Each method results in almost the same measure for angle B. The difference is due to the rounding-off procedures.

(8) 2. Use ΔABC to answer the following.



a. Find the measure of the unknown side to the nearest whole number.

Find a.

$$a^2 = c^2 - b^2$$
$$a^2 = 73^2 - 31^2$$
$$a^2 = 5329 - 961$$
$$a^2 = 4368$$
$$\sqrt{a^2} = \sqrt{4368}$$
$$a \doteq 66.091$$
$$a \doteq 66$$

The value for a is approximately 66.

b. Find the measure of both acute angles to the nearest degree. Use all three trigonometric ratios for each angle.

Find the measure of $\angle A$.

$$\sin A = \frac{66}{73}$$
$$\doteq 0.9041$$

To the nearest degree, $\angle A = 65°$.

$$\cos A = \frac{31}{73}$$
$$\doteq 0.4247$$

To the nearest degree, $\angle A = 65°$.

$$\tan A = \frac{66}{31}$$
$$\doteq 2.1290$$

To the nearest degree, $\angle A = 65°$.

Each method results in the same measure for $\angle A$.

Find the measure of $\angle B$.

$$\sin B = \frac{31}{73}$$
$$\doteq 0.4247$$

To the nearest degree, $\angle B = 25°$.

$$\cos B = \frac{66}{73}$$
$$\doteq 0.9041$$

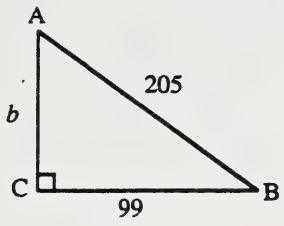
To the nearest degree, $\angle B = 25°$.

$$\tan B = \frac{31}{66}$$
$$\doteq 0.4697$$

To the nearest degree, $\angle B = 25°$.

Each method results in the same measure for $\angle B$.

(8) 3. Use ΔABC to answer the following.



a. Find the measure of the unknown side to the nearest whole number.

Find b.

$$b^2 = c^2 - a^2$$
$$b^2 = 205^2 - 99^2$$
$$b^2 = 42\,025 - 9801$$
$$b^2 = 32\,224$$
$$\sqrt{b^2} = \sqrt{32\,224}$$
$$b \doteq 179.51$$
$$b \doteq 180$$

The value for b is approximately 180 .

b. Find the measure of both acute angles to the nearest degree. Use all three ratios for each angle.

Find the measure for $\angle A$.

$$\sin A = \frac{99}{205}$$
$$\doteq 0.4829$$

To the nearest degree, $\angle A = 29°$.

$$\cos A = \frac{180}{205}$$
$$\doteq 0.8780$$

To the nearest degree, $\angle A = 29°$.

$$\tan A = \frac{99}{108}$$
$$= 0.5500$$

To the nearest degree, $\angle A = 29°$.

Each method results in the same measure for $\angle A$.

Find the measure of $\angle B$.

$$\sin B = \frac{180}{205}$$
$$\doteq 0.8780$$

To the nearest degree, $\angle B = 61°$.

$$\cos B = \frac{99}{205}$$
$$\doteq 0.4829$$

To the nearest degree, $\angle B = 61°$.

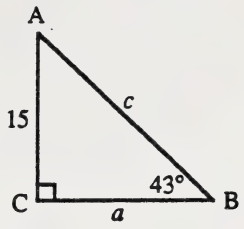
$$\tan B = \frac{180}{99}$$
$$\doteq 1.8182$$

To the nearest degree, $\angle B = 61°$.

Each method results in the same measure for $\angle B$.

(8) 4. Solve the indicated sides in each of the following triangles. Round each measure to the nearest whole number.

a.



Solve for c.

$$\sin 43° = \frac{15}{c}$$

$$0.6820 = \frac{15}{c}$$

$$c \doteq 15 \div 0.6820$$

$$c \doteq 22$$

The value for c is 22 rounded to the nearest whole number.

Solve for a.

$$\tan 43° = \frac{15}{a}$$

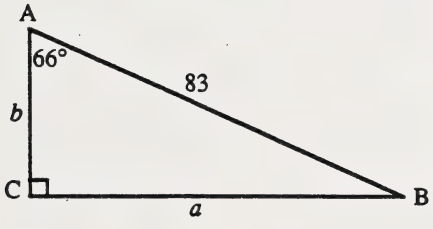
$$0.9325 = \frac{15}{a}$$

$$a = 15 \div 0.9325$$

$$a \doteq 16$$

The value for a is 16 rounded to the nearest whole number.

b.



Solve for a.

$$\sin 66° = \frac{a}{83}$$

$$0.9135 = \frac{a}{83}$$

$$a = 83 \times 0.9135$$

$$a \doteq 76$$

The value for a is 76 rounded to the nearest whole number.

Solve for b.

$$\cos 66° = \frac{b}{83}$$

$$0.4067 = \frac{b}{83}$$

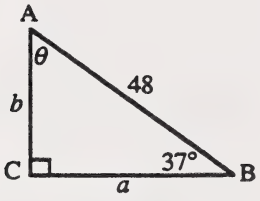
$$b = 83 \times 0.4067$$

$$b \doteq 34$$

The value for b is 34 rounded to the nearest whole number.

(6) 5. Solve each triangle. Provide a summary for each. Round all answers to the nearest whole number.

a.



$$\angle A = \theta = 180° - (90° + 37°)$$
$$\theta = 180° - 127°$$
$$\theta = 53°$$

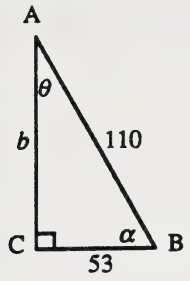
$$\sin 37° = \frac{b}{48}$$
$$0.6018 = \frac{b}{48}$$
$$b = 48 \times 0.6018$$
$$b \doteq 29$$

$$\cos 37° = \frac{a}{48}$$
$$0.7986 = \frac{a}{48}$$
$$a = 48 \times 0.7986$$
$$a \doteq 38$$

Summary

$\angle A = 53°$	$a = 38$
$\angle B = 37°$	$b = 29$
$\angle C = 90°$	$c = 48$

b.



$$\sin\theta = \frac{53}{110}$$
$$\doteq 0.4818$$
$$\theta = 29°$$
$$\angle A = \theta$$
$$= 29°$$

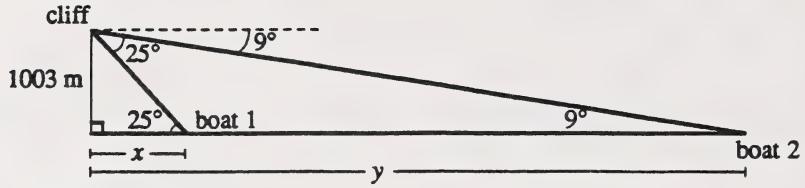
$$\cos\alpha = \frac{53}{110}$$
$$\doteq 0.4818$$
$$\alpha = 61°$$
$$\angle B = \alpha$$
$$= 61°$$

$$\tan\alpha = \frac{b}{53}$$
$$\tan 61° = \frac{b}{53}$$
$$1.8040 = \frac{b}{53}$$
$$b = 53 \times 1.8040$$
$$b \doteq 96$$

Summary:

$\angle A = 29°$	$a = 53$
$\angle B = 61°$	$b = 96$
$\angle C = 90°$	$c = 110$

④ 6. From the top of a vertical cliff 1003 m in height, the angles of depression of two boats due east are observed to be 25° and 9° respectively. What is the distance between the boats? Express the answer to the nearest metre.



Step 1: Solve for y.

$$\tan 9° = \frac{1003}{y}$$

$$0.1584 = \frac{1003}{y}$$

$$y = 1003 \div 0.1584$$

$$y \doteq 6332$$

Boat 2 is approximately 6332 m east of the cliff.

Step 2: Solve for x.

$$\tan 25° = \frac{1003}{x}$$

$$0.4663 = \frac{1003}{x}$$

$$x = 1003 \div 0.4663$$

$$x \doteq 2151$$

Boat 1 is approximately 2151 m east of the cliff.

Step 3: Subtract x from y.

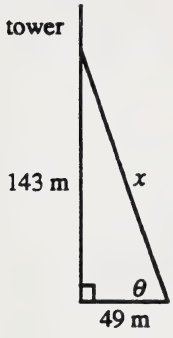
$y - x$ = distance between the two boats

$6332 - 2151 = 4181$

The distance between the two boats is approximately 4181 m.

④ 7. A guy wire is to be attached to a television transmission tower 143 m above the ground. The other end is to be fastened to the ground 49 m from the base of the tower.

a. What angle will the wire make with the ground? Express the answer to the nearest whole number.



Solve for θ.

$$\tan \theta = \frac{143}{49}$$

$$\tan \theta = 2.9184$$

$$\theta = 71°$$

The angle that the wire makes with the ground is approximately 71°.

b. What is the length of the wire? Express the answer to the nearest whole number.

Solve for x.

$$\sin \theta = \frac{143}{x}$$

$$\sin 71° = \frac{143}{x}$$

$$0.9455 = \frac{143}{x}$$

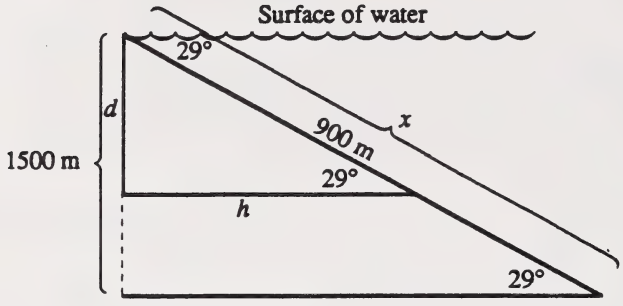
$$x = 143 \div 0.9455$$

$$x = 151$$

The length of the wire to the nearest metre is 151 m.

④ 8. A submarine at the surface of the ocean makes an emergency dive. Its path makes an angle of 29° with the surface.

a. If it goes 900 m along its downward path, how deep will it be? Express the answer to the nearest metre.



Solve for d, which is the depth after diving 900 m.

$$\sin 29° = \frac{d}{900}$$

$$0.4848 = \frac{d}{900}$$

$$d = 900 \times 0.4848$$

$$d \doteq 436$$

The submarine will be approximately 436 m deep after diving for 900 m.

b. What horizontal distance is it from its starting point? Express the answer to the nearest metre.

Solve for h which is the horizontal distance the submarine travelled from its starting point.

$$\cos 29° = \frac{h}{900}$$

$$0.8746 = \frac{h}{900}$$

$$h = 900 \times 0.8746$$

$$h \doteq 787$$

The submarine is approximately 787 m from its starting point in a horizontal direction.

c. How many metres must it go along its downward path to reach a depth of 1500 m? Express the answer to the nearest metre.

Solve for x which is the length of the dive to reach a depth of 1500 m.

$$\sin 29° = \frac{1500}{x}$$

$$0.4848 = \frac{1500}{x}$$

$$x = 1500 \div 0.4848$$

$$x = 3094$$

The submarine must dive approximately 3094 m to be at a depth of 1500 m.

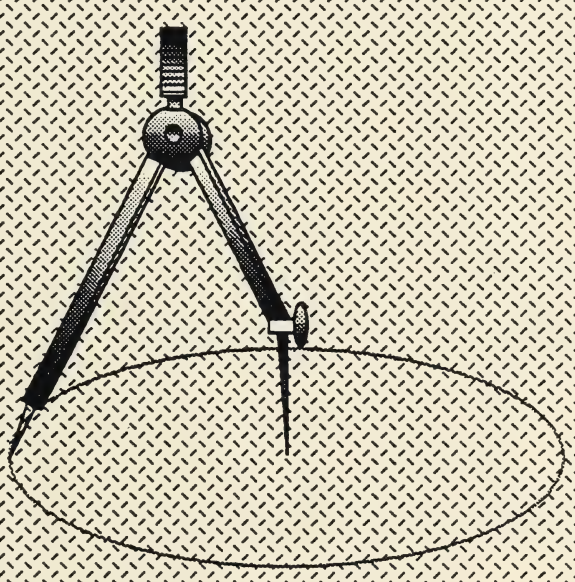
Topic 4

_____ marks

MATHEMATICS 23

LEARNING FACILITATOR'S MANUAL

UNIT 8



Distance Learning

Alberta EDUCATION

Topic 1: Probability

(5)

1. Conduct an experiment that involves choosing 3 balls at random from a box. (Make sure you record the colour and the order in which each ball was chosen.) The box contains 2 red balls, 1 green ball, and 2 blue balls. Since the order is important, at least 60 combinations will be required. From the recorded results of your experiment, determine the probability of choosing the combination blue–green–blue.

   Answers will vary. A sample experiment is given.

Combination	Tally	Combination	Tally	Combination	Tally
blue–red–blue	III	green–red–red	III	red–green–blue	II
green–blue–red	IIII	red–blue–red	IIIIIII	red–blue–blue	II
red–red–green	II	red–green–red	IIIIII	blue–red–red	IIII
blue–green–red	IIIIIII	blue–blue–red	III	blue–green–blue	II
green–red–blue	III	red–blue–green	II	red–red–blue	I
blue–red–green	IIIIIII	blue–blue–green	II	green–blue–blue	

   From the results of the experiment, $P(\text{blue–green–blue}) = \frac{2}{60} = \frac{1}{30}$.

(5)

2. Conduct an experiment that involves tossing a coin into the air 4 times and recording, in order, how it lands. Repeat the experiment at least 20 times. From the results of your experiment, determine the probability of obtaining two heads and two tails.

   Answers will vary. A sample experiment is given.

Combination	4H	3H, 1T	2H, 2T	1H, 3T	4T
Tally	I	IIIIII	IIIII	IIIIIII	I

   From the results of the experiment, $P(2\,\text{H},\ 2\text{T}) = \frac{5}{20} = \frac{1}{4}$.

(5)

3. Conduct an experiment that involves using only the kings and the queens from a deck of playing cards. From these 8 cards, deal 2 cards to one player and 3 cards to another. Repeat the experiment at least 30 times. From the results of your experiment, determine the probability of dealing the king of hearts and a black queen to the player with 2 cards.

Answers will vary. A sample experiment is given.

Trial	Combination 1st	2nd	Trial	Combination 1st	2nd	Trial	Combination 1st	2nd
1	Q♥	Q♣	11	K♣	Q♣	21	K♥	Q♥
2	K♣	Q♣	12	Q♠	K♥	22	K♠	K♦
3	K♣	Q♦	13	K♦	K♥	23	K♥	K♣
4	Q♣	K♣	14	Q♠	K♦	24	K♦	Q♦
5	Q♦	K♦	15	K♠	K♣	25	K♠	Q♠
6	K♣	Q♣	16	Q♠	K♥	26	K♦	Q♣
7	K♠	Q♥	17	K♥	K♦	27	Q♣	Q♦
8	Q♦	K♦	18	K♥	K♠	28	Q♦	Q♥
9	Q♥	Q♦	19	Q♣	K♠	29	Q♣	Q♦
10	K♣	K♥	20	K♠	K♥	30	K♠	Q♠

Trial	Combination 1st	2nd
1	Q♥	K♦K♣Q♦
2	K♣	K♠K♦Q♥
3	K♣	K♥Q♣K♦
4	Q♣	K♥K♦K♠
5	Q♦	Q♣K♥K♠
6	K♣	K♠K♦K♥
7	K♠	K♥K♣Q♠
8	Q♦	K♠K♣K♥
9	Q♥	K♣K♠K♥
10	K♣	Q♦K♦Q♥

Trial	Combination 1st	2nd
11	K♣	K♦Q♥K♠
12	Q♠	Q♣Q♦Q♥
13	K♦	Q♠Q♣K♣
14	Q♠	K♥Q♣Q♥
15	K♠	K♥Q♦Q♥
16	Q♠	K♣Q♥K♦
17	K♥	K♣Q♠Q♥
18	K♥	Q♥Q♣K♦
19	Q♣	Q♥Q♠K♦
20	K♠	Q♥K♣Q♣

Trial	Combination 1st	2nd
21	K♥	Q♥ K♣Q♣Q♠
22	K♠	K♦ Q♠Q♣K♥
23	K♥	K♣ Q♥K♦Q♦
24	K♦	Q♦ Q♥Q♠K♠
25	K♠	Q♦ Q♠Q♣K♦
26	K♦	Q♣ Q♦K♠Q♥
27	Q♣	Q♦ K♦Q♥K♣
28	Q♦	Q♥ K♦K♥K♠
29	Q♣	Q♦ K♥Q♥Q♠
30	K♠	Q♦ K♣Q♥Q♣

From the experiment, $P(K\heartsuit, Q_{\text{black}}) = \frac{2}{30} = \frac{1}{15}$.

(5) 4.

Number	Frequency Tally
2	II
3	IIII
4	IIIIIII
5	IIIIII
6	IIIIIIII
7	IIIIIIIIIIIII
8	IIIIIIIIII
9	IIIIII
10	IIIIII
11	III
12	I

Two dice are rolled as part of a probability experiment and the results are recorded above. From the frequency table, determine the following. Express these as reduced fractions.

a. the probability of rolling a number greater than 4

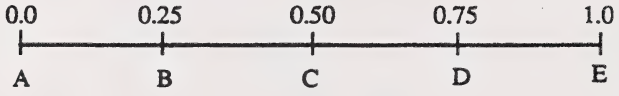
Probability of rolling a number > 4 = $\frac{54}{67}$.

b. the probability of rolling a number less than or equal to 10

Probability of rolling a number ≤ 10 = $\frac{63}{67}$.

(5) 5. Circle the correct response.

PROBABILITY LINE

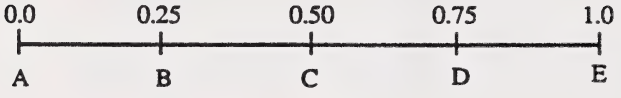


If an event is **certain** to occur, which probability represents this situation?

a. A
b. B
c. C
d. D
e. E

(5) 6. Circle the correct response.

PROBABILITY LINE



If the probability that an event will occur is equal to the probability that it will not occur, which position on the probability line represents this situation?

a. A
b. B
c. C
d. D
e. E

(5) 7. Suppose you are tossing a coin into the air and recording whether it lands heads (H) or tails (T). If you toss the coin thrice, what is the probability of recording HTT? Express your answer as a ratio in reduced fractional form.

There is only 1 way to get HTT.
There are 8 different outcomes.
(The order of tosses are important.)

P(HTT) = $\frac{1}{8}$

(5) 8. Two dice are tossed. What is the probability of tossing a number less than or equal to 10? Express your answer as a ratio in reduced fractional form.

There are 33 ways to get 10 or less.
There are 36 possible results on the dice.

P(number ≤ 10) = $\frac{33}{36}$

P(number ≤ 10) = $\frac{11}{12}$

(5) 9. A basket at the fruit stand contains 21 apples, 19 oranges, and 8 plums. If you were to close your eyes and reach into the basket to select one piece of fruit, what would be the probability of not purchasing a plum? Express your answers as a ratio reduced to lowest terms.

There are 21 + 19 or 40 items that are not plums.
There are 21 + 19 + 8 or 48 items in the basket.

P(not plum) = $\frac{40}{48}$

P(not plum) = $\frac{5}{6}$

There is a probability of $\frac{5}{6}$ of not selecting a plum.

(5) 10.

Demographic Report for Huntsville

Age Class (a)	Number of People
0 - 9	156
10 - 19	194
20 - 29	152
30 - 39	155
40 - 49	124
50 - 59	107
60 - 69	74
70 +	38

While on a tour of Huntsville, suppose you said good-bye to the last person you saw. Expressing your answer in decimal form, determine the probability that the person is in one of the following categories.

a. older than 39 years of age

$$P(>39) = \frac{124+107+74+38}{1000}$$

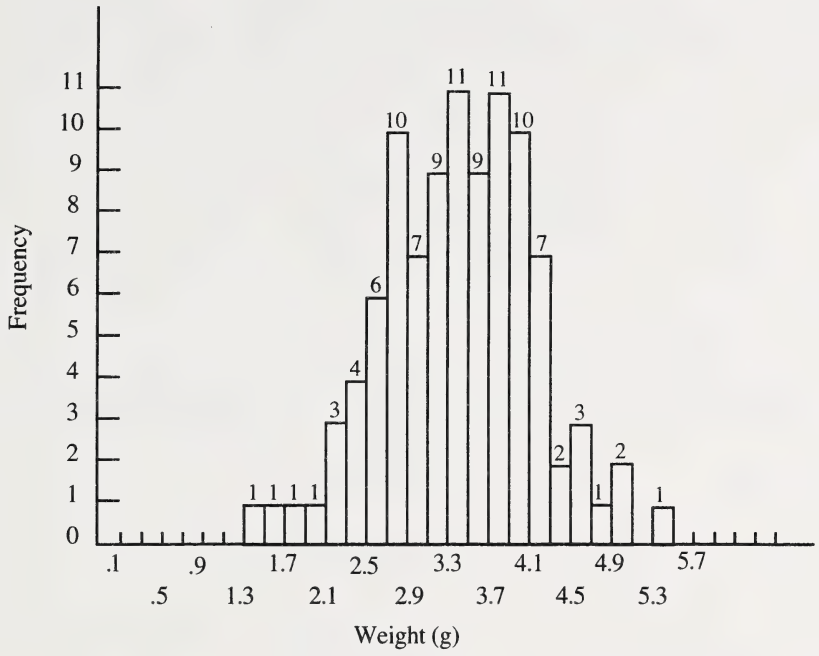
$$= \frac{343}{1000} \text{ or } 0.343$$

b. less than 50 years of age

$$P(<50) = \frac{124+155+152+194+156}{1000}$$

$$= \frac{781}{1000} \text{ or } 0.781$$

(6) 11.



The histogram above represents the weights and numbers in stock of certain chocolate bars. You decide to buy some chocolate bars at random. The probability of chocolate bars of weight between 2.1 and 2.9 grams is given by

$$P(2.1 < m < 2.9) = \frac{3+4+6}{100} = \frac{13}{100} = 13\% \text{ (or } 0.13).$$

Find the probability of

a. 1.7 g and 2.5 g

$$P(1.7 < m < 2.5) = \frac{1+1+3+4}{100}$$
$$= \frac{9}{100} \text{ or } 0.09$$

b. 2.9 g and 3.7 g

$$P(2.9 < m < 3.7) = \frac{7+9+11+9}{100}$$
$$= \frac{9}{25} \text{ or } 0.36$$

c. 4.1 g and 5.7 g

$$P(4.1 < m < 5.7) = \frac{7+2+3+1+2+1}{100}$$
$$= \frac{4}{25} \text{ or } 0.16$$

(6) 12.

7	8	10	9
11	8	6	12
8	7	12	8
6	5	10	9
7	8	8	7
10	11	8	9
9	8	7	9
10	7	9	8

The above chart represents the shoe sizes of the students in Ms. Hunt's dance class. One student is selected at random to perform a tap dance. Expressing your answers as reduced fractions, determine the probability that the student's shoe size will be in one of the following categories.

a. size 9 $P(9) = \frac{6}{32}$

$= \frac{3}{16}$

b. either size 8 or size 9 $P(8 \text{ or } 9) = \frac{15}{32}$

c. not less than size 8 $P(\geq 8) = \frac{23}{32}$

(6) 13. As part of a compound event, three coins are tossed in the air and a record is made of how they land. Expressing your answers as simplified fractions, determine the probability of getting one of the following:

a. 2 tails $P(2T) = \frac{3}{8}$ Desirable outcomes

HTT, THT, TTH

b. 0 tails $P(0T) = \frac{1}{8}$ Desirable outcomes

HHH

c. 3 heads $P(3H) = \frac{1}{8}$ Desirable outcomes

HHH

There are 8 possible outcomes.

(6) 14. As part of a compound event, three cards are drawn from a deck of 52 playing cards. Expressing your answers as reduced fractions, determine the probability of drawing the following combinations.

a. the ace ♣, king ♣, and queen ♣

$$P(A\clubsuit, K\clubsuit, Q\clubsuit) = \frac{3}{52} \times \frac{2}{51} \times \frac{1}{50}$$

$$= \frac{6}{132\,600}$$

$$= \frac{1}{22\,100}$$

b. the 7 ♠, 5 ♣, and 6 ♠

$$P(7\spadesuit, 5\clubsuit, 6\spadesuit) = \frac{3}{52} \times \frac{2}{51} \times \frac{1}{50}$$

$$= \frac{6}{132\,600}$$

$$= \frac{1}{22\,100}$$

(5) 15. Two items are selected at random, first from Table 1 and then from Table 2. Determine the following probabilities of the compound event. Report your answers as reduced fractions.

Table 1

4	2	4	1
1	3	1	2
3	2	1	2
2	3	4	1
1	2	4	4
1	3	4	1
1	2	4	2

Table 2

★	○	∞	Δ
∞	○	∞	★
★	Δ	★	Δ
★	○	★	○
★	∞	∞	★
∞	○	Δ	Δ
Δ	○	∞	∞

a. Probability (2 and ○)

$$P(2 \text{ and } ○) = \frac{8}{28} \times \frac{6}{28}$$

$$= \frac{48}{784}$$

$$= \frac{3}{49}$$

b. Probability (3 and Δ)

$$P(3 \text{ and } Δ) = \frac{4}{28} \times \frac{6}{28}$$

$$= \frac{24}{784}$$

$$= \frac{3}{98}$$

(6) 16. Loretta decides that she will simply guess at the answers to a 10-question true/false test. Before she writes the exam she simulates the situation using a coin and by letting **heads** represent a correct response. The following table is a summary of the outcomes of her simulation for the number of heads occurring per trial of 10 coin tosses.

Number of Heads	Number of Trials	Estimated Probability
0	2	$\frac{1}{72}$
1	3	$\frac{1}{48}$
2	9	$\frac{1}{16}$
3	17	$\frac{17}{144}$
4	28	$\frac{7}{36}$
5	34	$\frac{17}{72}$
6	26	$\frac{13}{72}$
7	13	$\frac{13}{144}$
8	7	$\frac{7}{144}$
9	5	$\frac{5}{144}$
10	0	0

Complete the above table and determine the probability that she will obtain the following scores. Express your answers as reduced fractions.

a. not less than 60%

$$P(\not< 60\%) = \frac{13}{72} + \frac{13}{144} + \frac{7}{144} + \frac{5}{144} + 0$$
$$= \frac{26+13+7+5+0}{144}$$
$$= \frac{51}{144}$$
$$= \frac{17}{48}$$

b. between 30% and 70% (inclusive)

$$P(30\% \le s \le 70\%) = \frac{17}{144} + \frac{7}{36} + \frac{17}{72} + \frac{13}{72} + \frac{13}{144}$$
$$= \frac{17+28+34+26+13}{144}$$
$$= \frac{118}{144}$$
$$= \frac{59}{72}$$

(5) 17. Circle the correct response.
Determine the most appropriate simulation for the following situation.
Mary has two pairs of shoes to choose from to wear to school. She randomly selects a pair of shoes each morning. What is the probability that she wears the same pair of shoes for a week?

A. Use a random number table considering odd and even numbers only.

B. Draw 2 numbers from a hat.

C. Toss a coin.

(D.) All of the above.

(5) 18. Circle the correct response.
Determine the best simulation for the following situation.
Mary has five pairs of shoes to choose from to wear to school. She randomly selects a pair of shoes each morning. What is the probability that she wears the same pair of shoes for six consecutive days?

A. Use a random number table of 6-digit numbers.

B. Roll a die.

C. Toss a coin 3 times.

(D.) All of the above.

(5) 19. In many countries, it is the son who takes care of the parents when they have reached their old age. Because of this, couples are inclined to continue to have children until they have a son.

a. Using this situation as the rule, do a simulation to find the following information.

i. the average number of children per family

Solutions will vary. This is one possible simulation.

B	B	GGGB	GB	B
GGGGB	GGB	GGB	B	GGB
B	GGGB	B	B	B
B	B	GGB	B	B
B	B	GB	GGGB	GB
GGB	GB	GGB	GB	GB
B	B	GGB	B	GB
GGGGGGGB	GGB	GGGB	GB	B
B	GB	B	GB	GGGB
GGB	B	B	GGGGB	GB

$\frac{109}{50}$ or about 2

There will be an average of 2 children per family.

ii. the average number of girls per family

Solutions will vary.

$\frac{59}{50}$ or about 1

There will be an average of 1 girl per family.

b. Would you expect more girls or more boys to be born? Explain your answer.

The answers will vary. Check to see how they support their answer.

Topic 1

_____ marks

Final Test

There are two copies of the final test: the student's copy which is perforated and designed for photocopying and possible faxing and the teacher's copy which includes a marking guide.

Note:

The student's copy of this final test should be kept secure by the teacher. Students should not have access to this test until it is assigned in a supervised situation. The answers should be stored securely and retained by the teacher at all times.

MATHEMATICS 23

FINAL TEST

GENERAL INSTRUCTIONS

1. The time limit for this test is **2 hours**.

2. There are **150 marks** on this exam.

   PART A: Multiple Choice - 30 marks

   PART B: Written Response - 120 marks

3. Calculators that are not programmed may be used.

4. Trigonometric tables are provided at the end of the test.

5. It is better to complete all the work you know well first. After that you can deal with the difficult ones. Do not spend too much time on any one problem. If you finish the test before the two-hour time limit, go back and check your answers.

PART A: MULTIPLE CHOICE

All multiple-choice questions must be answered on the Part A Response Page included in your test.

Read carefully and decide which of the choices BEST completes the statement or answers the question. Locate the question number on the Response Page and place the appropriate letter A, B, C, D, or E in the blank.

Example:

83. The value of -5^2 is

A. 10

B. -10

C. 25

D. -25

E. none of the above

Response Page

D 83.

1. The value of $\dfrac{(-27)^{\frac{4}{3}}}{(64)^{-\frac{1}{3}}}$ is

A. $\dfrac{16}{81}$

B. $\dfrac{81}{4}$

C. 81

D. 324

E. none of the above

2. The G.C.F. of $15x^3y^2z$, $30x^2y^2z^3$, and $60x^4y^3z^2$ is

   A. $60x^4y^3z^3$

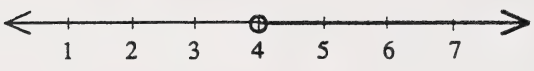
   B. $15x^2y^2z$

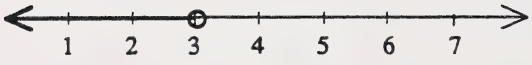
   C. $30x^4y^3z^3$

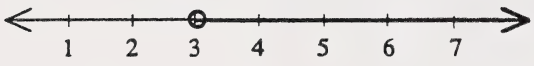
   D. $60x^2y^2z$

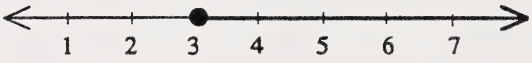
   E. none of the above

3. The graph of the solution to the inequality $-5x+7<-8$ is

   A. 

   B. 

   C. 

   D. 

   E. none of the above

4. The straight-line graph of $y = -4$ has the characteristic that it

   A. is parallel to the y-axis

   B. has an x-intercept

   C. has no y-intercept

   D. has a slope of 0

   E. none of the above

5. The slope of a line perpendicular to the graph of $y = 7x + 3$ is

   A. 7

   B. -7

   C. $\frac{1}{7}$

   D. $-\frac{1}{7}$

   E. none of the above

6. A linear equation system in which the slopes of the two equations are the same and the y-intercepts are the same has

   A. no solution

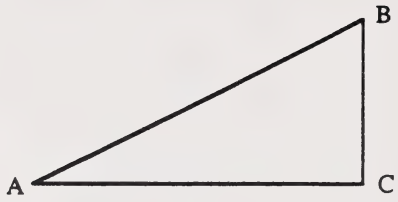
   B. one solution

   C. two solutions

   D. an infinite number of solutions

   E. none of the above

7. With respect to $\angle A$, the trigonometric ratio $\frac{BC}{AC}$ is called the



A. tangent of $\angle A$

B. cosine of $\angle A$

C. sine of $\angle A$

D. secant of $\angle A$

E. none of the above

8. The formula $A = 4\pi r^2$ finds the surface area of a

A. cone

B. cylinder

C. dome

D. sphere

E. none of the above

9. If $\sin\theta = \frac{\sqrt{3}}{2}$, what is $\cos\theta$?

A. 2

B. $\frac{1}{2}$

C. $\sqrt{3}$

D. $\frac{2}{\sqrt{3}}$

E. none of the above

10. The probability of rolling a 2, 3, or 4 using a die is

A. $\frac{3}{5}$

B. $\frac{2}{3}$

C. $\frac{1}{2}$

D. $\frac{3}{4}$

E. none of the above

11. The value of $4^{-2}\left(\frac{1}{5}\right)^{-2}(-2)^{-1}$ is

A. $\frac{25}{32}$

B. $-\frac{25}{32}$

C. $\frac{32}{25}$

D. $-\frac{32}{25}$

E. none of the above

12. The amount paid on a $9200 loan at the end of 5 years at 8% per annum compounded semiannually is approximately

A. $12 967.16

B. $13 053.32

C. $13 412.76

D. $13 618.25

E. none of the above

13. Suppose you toss a coin and record whether it lands heads (H) or tails (T). If you toss the coin 3 times, the probability of recording HTH is

A. $\frac{1}{9}$

B. $\frac{1}{8}$

C. $\frac{1}{6}$

D. $\frac{1}{3}$

E. none of the above

14. Determine which statement is NOT true.

A. The area of a trapezoid is $\frac{1}{2}h(a+b)$.

B. The volume of a sphere with a radius of 3 cm is $113\ \text{cm}^3$.

C. A pyramid is a solid that has a polygon as a base.

D. Euler's formula is $V + E - F = 2$.

E. none of the above

15. The solution to the linear system $5x - 2y = -19$ and $3x + 3y = -3$ is

A. $(3, -2)$

B. $(3, 2)$

C. $(-3, 2)$

D. $(-3, -2)$

E. none of the above

PART A: RESPONSE PAGE

D	1.	B	9.
B	2.	C	10.
C	3.	B	11.
D	4.	D	12.
D	5.	B	13.
D	6.	D	14.
A	7.	C	15.
D	8.		

Value

PART B: WRITTEN RESPONSE

Be sure to show all your work. Part marks will be given for partially correct solutions.

1. Simplify each of the following.

(2) a. $(10^4)^2 \times 6^0$

$= 10^8 \times 1$

$= 10^8$

(2) b. $\frac{10}{6^{-2}}$

$= 10 \times 6^2$

$= 10 \times 36$

$= 360$

(3) c. $\sqrt[3]{\frac{250}{16}}$

$= \sqrt[3]{\frac{125}{8}}$

$= \frac{5}{2}$

(2) d. $343^{\frac{2}{3}}$

$= 7^2$

$= 49$

(2) e. $\left(\frac{x^2y^2}{d^4} \times \frac{d^3xy^3}{y^4}\right)^2$

$= \left(\frac{x^3y^5}{dy^4}\right)^2$

$= \left(\frac{x^3y}{d}\right)^2$

$= \frac{x^6y^2}{d^2}$

Value

2. Factor fully.

(1) a. $8x^2y^3-24x^2y^5$

$=8x^2y^3(1-3y^2)$

(2) b. $2x(x-6y)-7(x-6y)$

$=(2x-7)(x-6y)$

(2) c. $(x+y)^2-16$

$=(x+y+4)(x+y-4)$

(2) d. $8x^2+10x-3$

$=(4x-1)(2x+3)$

(2) 3. Solve $8-3x \leq -7$.

$8-3x \leq -7$

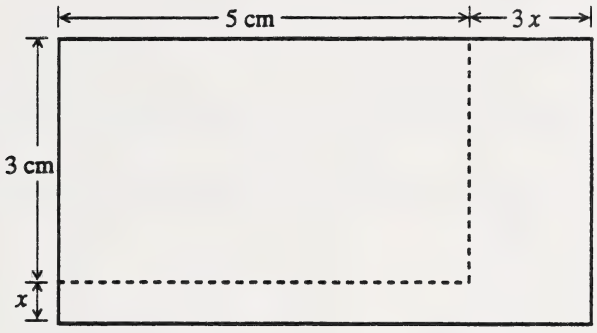
$-3x \leq -7-8$

$-3x \leq -15$

$x \geq 5$

Value

(5) 4. A rectangular piece of metal is increased by x cm in one dimension and $3x$ cm in the other dimension. Determine the new dimensions of the metal if the new plate has an area of $84\ \text{cm}^2$.



$$(3+x)(5+3x)=84$$

$$15+5x+9x+3x^2=84$$

$$3x^2+14x-69=0$$

$$(3x+23)(x-3)=0$$

$$3x+23=0 \qquad x-3=0$$

$$x=-\frac{23}{3} \qquad x=3$$

The dimensions are $(3 + x)$ by $(5 + 3x)$

$(3 + 3)$ by $[(5 + 3\ (3)]$

6 by 14

The dimensions are 6 cm × 14 cm.

TEACHER'S COPY

Value

(4) 5. How much money must be invested today at 9% per annum compounded semiannually in order to produce $6800 in 7 years?

$$A = P(1+i)^n$$
$$6800 = P(1.045)^{14}$$
$$P = \frac{6800}{(1.045)^{14}}$$
$$P = \$3671.82$$

(2) 6. Determine the slope of the line through A(– 6, 5) and B(– 2, 9).

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{9-5}{-2-(-6)}$$
$$= \frac{4}{-2+6}$$
$$= \frac{4}{4}$$
$$= 1$$

(3) 7. Write the equation $7x - 3y - 6 = 0$ in the form $y = mx + b$ and state the slope and the y-intercept.

$$7x - 3y - 6 = 0$$
$$-3y = -7x + 6$$
$$y = \frac{-7x}{-3} + \frac{6}{-3}$$
$$y = \frac{7}{3}x - 2$$

slope $= \frac{7}{3}$

y-intercept $= -2$

TEACHER'S COPY

Value

(4) 8. Determine the equation of the line that is parallel to $x = -3$ and passes through the point $(4, -8)$.

$x = 4$

(5) 9. Write the equation of the line that is perpendicular to $y = -5x + 7$ and passes through the point $(2, -8)$. Write the equation in the general form $ax + by + c = 0$.

$$y = -5x + 7$$
$$\text{slope} = -5$$

The slope of the required line is $\frac{1}{5}$.

$$\frac{y-(-8)}{x-2} = \frac{1}{5}$$
$$\frac{y+8}{x-2} = \frac{1}{5}$$
$$5(y+8) = 1(x-2)$$
$$5y+40 = x-2$$
$$x-5y-42 = 0$$

Value

(3) 10. The sum of two numbers is 32. Two times the smaller number is 10 more than the larger number. Determine the two numbers.

$$\begin{aligned} x+y&=32 \quad (1)\\ 2x-y&=10 \quad (2)\\ \hline 3x&=42 \quad (1)+(2)\\ x&=14 \end{aligned}$$

$$\begin{aligned} x+y&=32 \quad (1)\\ 14+y&=32\\ y&=18 \end{aligned}$$

(5) 11. A rectangular field has a perimeter of 4080 m. The length is 240 m less than twice the width. Determine the length and width of the field. Use a final statement in your answer.

Let l = length
and w = width.

$$\begin{aligned} 2w+2l&=4080 \quad (1)\\ 2w-l&=240 \quad (2)\\ \hline 3l&=3840 \quad (1)-(2)\\ l&=1280\\ 2w-1280&=240 \quad (2)\\ 2w&=1280+240\\ 2w&=1520\\ w&=760 \end{aligned}$$

The width is 760 m and the length is 1280 m.

TEACHER'S COPY

Value

(6)

12. Solve the system.

$$6x-4y=3$$
$$9x+8y=1$$

$$\begin{aligned} 6x-4y&=3 && (1)\times 2\\ 9x+8y&=1 && (2)\end{aligned}$$

$$\begin{aligned} 12x-8y&=6 && (3)\\ \underline{9x+8y}&\underline{=1} && (2)\\ 21x&=7 && (3)+(2)\end{aligned}$$

$$x=\frac{7}{21}$$

$$x=\frac{1}{3}$$

$$\begin{aligned} 6\left(\frac{1}{3}\right)-4y&=3 && (1)\\ 2-4y&=3\\ -4y&=3-2\\ -4y&=1\\ y&=-\frac{1}{4}\end{aligned}$$

$$x=\frac{1}{3},\ y=-\frac{1}{4}$$

Value

(6) 13. Four beefburgers and three glasses of milk cost \$8.50. Two beefburgers and one glass of milk cost \$3.90. Determine the cost of one beefburger and the cost of one glass of milk.

Let x = cost of one beefburger.
Let y = cost of one glass of milk.

$$4x+3y=8.50 \quad (1)$$
$$2x+1y=3.90 \quad (2) \times -2$$

$$4x+3y=8.50 \quad (1)$$
$$-4x-2y=-7.80 \quad (3)$$
$$y=0.70 \quad (1)+(3)$$

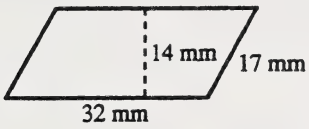
$$2x+1y=3.90 \quad (2)$$
$$2x+0.70=3.90$$
$$2x=3.20$$
$$x=1.60$$

One beefburger costs \$1.60 and one glass of milk costs 70¢.

Value

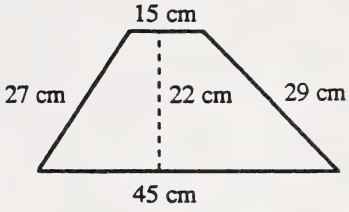
(10) 14. Calculate the following areas.

a.



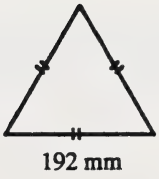
$$\begin{aligned}\text{Area} &= \text{base} \times \text{height} \\ &= 32 \text{ mm} \times 14 \text{ mm} \\ &= 448 \text{ mm}^2\end{aligned}$$

b.



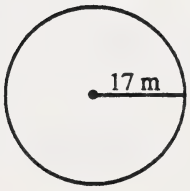
$$\begin{aligned}\text{Area} &= \frac{1}{2} \times \text{height}(\text{base} + \text{base}) \\ &= \frac{1}{2} \times 22 \text{ cm}(15 \text{ cm} + 45 \text{ cm}) \\ &= \frac{1}{2} \times 22 \text{ cm}(60 \text{ cm}) \\ &= 660 \text{ cm}^2\end{aligned}$$

c.



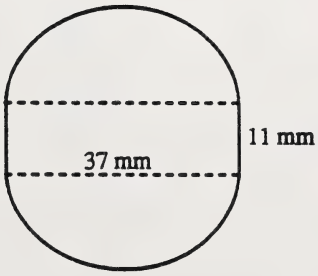
$$\begin{aligned}\text{Area} &= \frac{\sqrt{3}}{4} \times \text{side}^2 \\ &= \frac{\sqrt{3}}{4} \times (192 \text{ mm})^2 \\ &\doteq 15\,963 \text{ mm}^2\end{aligned}$$

d.



$$\begin{aligned}\text{Area} &= \pi \times \text{radius}^2 \\ &= \pi(17 \text{ m})^2 \\ &= \pi \times 289 \text{ m}^2 \\ &\doteq 907.9 \text{ m}^2\end{aligned}$$

e.

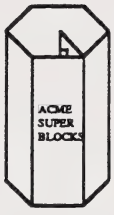


$$\begin{aligned}\text{Area} &= \text{Area circle} + \text{Area rectangle} \\ &= \pi \times \text{radius}^2 + \text{length} \times \text{width} \\ &= \pi\left(\frac{37 \text{ mm}}{2}\right)^2 + 37 \text{ mm} \times 11 \text{ mm} \\ &= \pi \times 342.25 \text{ mm}^2 + 407 \text{ mm}^2 \\ &\doteq 1075 \text{ mm}^2 + 407 \text{ mm}^2 \\ &\doteq 1482 \text{ mm}^2\end{aligned}$$

TEACHER'S COPY

Value

(5) 15. What volume will be displaced by a regular-hexagonal prism that is used for packaging toy building blocks? Each side of the hexagon is 8 cm. The radius of the hexagon is 7 cm. The height of the container is 22 cm.



$$\text{Volume} = \text{Area of base} \times \text{height}$$
$$= 6 \times \text{radius} \times \frac{1}{2}\text{base} \times \text{height}$$
$$= 6 \times 7 \text{ cm} \times 4 \text{ cm} \times 22 \text{ cm}$$
$$= 3696 \text{ cm}^3$$

The prism will displace 3696 cm^3.

(5) 16. What is the diameter of the moon Europa if it has a surface area of 30 270 000 km^2?

$$\text{Surface Area} = 4\pi \times \text{radius}^2$$
$$30\,270\,000 \text{ km}^2 = 4\pi \times \text{radius}^2$$

$$\frac{30\,270\,000 \text{ km}^2}{4\pi} = \text{radius}^2$$

$$\sqrt{\frac{30\,270\,000 \text{ km}^2}{4\pi}} = \text{radius}^2$$

$$\sqrt{2\,408\,810 \text{ km}^2} \doteq \text{radius}$$

$$1552 \text{ km} \doteq \text{radius}$$

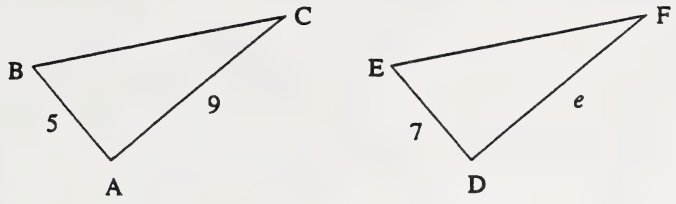
$$\text{diameter} = 2 \times \text{radius}$$
$$\text{diameter} = 2 \times 1552 \text{ km}$$
$$\text{diameter} = 3104 \text{ km}$$

Europa has a diameter of 3104 km.

TEACHER'S COPY

Value

(4) 17. Given $\Delta ABC \sim \Delta DEF$, determine the value of e to one decimal place.



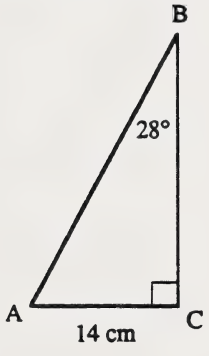
$$\frac{5}{9} = \frac{7}{e}$$

$$5e = 9(7)$$

$$e = \frac{9(7)}{5}$$

$$e = 12.6$$

(5) 18. Given a right triangle ABC, where $\angle B = 28°$ and side $b = 14$ cm, determine the length of the hypotenuse of the triangle. Round the final answer to two decimal places.



$$\sin 28° = \frac{14}{AB}$$

$$AB = \frac{14}{\sin 28°}$$

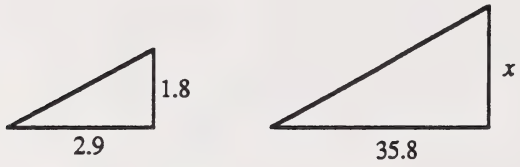
$$= \frac{14}{0.469\,47}$$

$$AB \doteq 29.82 \text{ cm}$$

TEACHER'S COPY

Value

(5) 19. A post 1.8 m tall casts a shadow 2.9 m along the ground. At the same time, a building has a shadow 35.8 m long. Determine the height of the building to one decimal place.



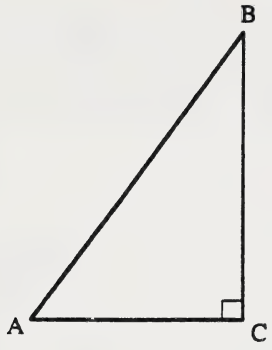
$$\frac{1.8}{2.9} = \frac{x}{35.8}$$

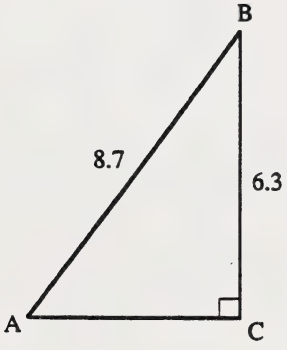
$$x = \frac{1.8(35.8)}{2.9}$$

$$x = 22.2 \text{ m}$$

Value

(6) 20. Solve ΔABC if $\angle C = 90°$, BA = 8.7 cm, and BC = 6.3 cm. Round the angle measurements to a whole number and the side measurement to one decimal place.





$$\sin A = \frac{6.3}{8.7}$$
$$= 0.7241$$
$$\angle A = 46°$$
$$\angle B = 90° - 46°$$
$$= 44°$$

$$\cos A = \frac{AC}{AB}$$
$$\cos A = \frac{AC}{8.7}$$
$$AC = 8.7 \cos A$$
$$= 8.7 \cos 46°$$
$$AC \doteq 6.0 \text{ cm}$$

TEACHER'S COPY

Value

(5) 21. There are 15 cards of equal size. Each card has a number from 1 to 15. The cards are placed in a hat and mixed. Determine the theoretical probabilities for the following situations.

a. What is the probability of drawing a number greater than 11?

The possible numbers are 12, 13, 14, 15. $P(>11)=\frac{4}{15}$

b. What is the probability of drawing an odd number?

The possible numbers are 1, 3, 5, 7, 9, 11, 13, 15. $P(\text{odd})=\frac{8}{15}$

c. What is the probability of drawing a prime number?

The possible numbers are 2, 3, 4, 7, 11, 13. $P(\text{prime})=\frac{6}{15}$

$=\frac{2}{5}$

d. What is the probability of drawing a number greater than 6 but less than 12?

The possible numbers are 7, 8, 9, 10, 11. $P(6<x<12)=\frac{5}{15}$

$=\frac{1}{3}$

e. What is the probability of drawing a number less than 6 and greater than 10?

This is an impossible case. $P(10<x<6)=\frac{0}{15}$

$=0$

Value

22. Use the information in the table to answer the following questions.

Math Marks	
Mark (%)	Frequency
0 - 40	4
41 - 45	6
46 - 50	5
51 - 55	7
56 - 60	8
61 - 65	12
66 - 70	13
71 - 75	12
76 - 80	9
81 - 85	8
86 - 90	2
91 - 95	2
96 - 100	1
Total	89

(1) a. What is the probability that a mark was above 90%?

$$P(>90\%) = \frac{3}{89}$$
$$\doteq 0.033$$

(2) b. What is the probability that a mark was from 51 to 75?

$$P(51 \text{ to } 75) = \frac{52}{89}$$
$$\doteq 0.584$$

(2) c. A mark over 50 but less than 66 is a C standing. What percent of the students receive C's?

$$P(50 \text{ to } 66) = \frac{27}{89}$$
$$\doteq 0.303$$

(2) d. If 118 students wrote the exam, determine how many students would receive a mark from 66 to 80%.

$$P(66 \text{ to } 80) = \frac{34}{89}$$
$$\doteq 0.382$$
$$0.382 \times 118 = 45.07$$
$$\doteq 45$$

TEACHER'S COPY

Value

23. An office worker has ten different hats. This worker decides to randomly select a hat to wear to the office each day for the five-day work week. Use a simulation to answer the following questions. Use the random number table of 50 trials given below and let the digits 0 to 9 each represent a different hat. Each block of 5 will represent a five-day week. To answer the questions, each trial must be checked.

52097	96239	72255	86588	13395	60424	63156	92309	79545	64637
58233	53891	61336	38086	39495	37774	27949	34573	58358	54006
10884	42540	17696	86020	78016	38734	04767	49518	19256	24990
35593	83183	91244	68292	24406	06991	47542	18392	25866	76750
67050	00008	52103	59222	26126	74210	01129	15483	86292	53468

(3) a. Estimate the probability that this worker will wear a different hat each day.

There are 10 blocks that have five different numbers.

approximately $\frac{10}{50}$

(1) b. Estimate the probability that this worker will wear the same hat each day.

There is no trial where a digit repeats 5 times.

approximately 0

(1) c. Estimate the probability that this worker will wear the same hat more than once in one week.

approximately $1-\frac{10}{50}=\frac{40}{50}=\frac{4}{5}$

TABLE OF TRIGONOMETRIC FUNCTIONS

ANGLE deg	sin	cos	tan	csc	sec	cot
0	0.0000	1.0000	0.0000	undefined	1.0000	undefined
1	0.0175	0.9998	0.0175	57.299	1.0002	57.2900
2	0.0349	0.9994	0.0349	28.654	1.0006	28.6363
3	0.0523	0.9986	0.0524	19.107	1.0014	19.0811
4	0.0698	0.9976	0.0699	14.336	1.0024	14.3007
5	0.0872	0.9962	0.0875	11.474	1.0038	11.4301
6	0.1045	0.9945	0.1051	9.5668	1.0055	9.5144
7	0.1219	0.9925	0.1228	8.2055	1.0075	8.1443
8	0.1392	0.9903	0.1405	7.1853	1.0098	7.1154
9	0.1564	0.9877	0.1584	6.3925	1.0125	6.3138
10	0.1736	0.9848	0.1763	5.7588	1.0154	5.6713
11	0.1908	0.9816	0.1944	5.2408	1.0187	5.1446
12	0.2079	0.9781	0.2126	4.8097	1.0223	4.7046
13	0.2250	0.9744	0.2309	4.4454	1.0263	4.3315
14	0.2419	0.9703	0.2493	4.1336	1.0306	4.0108
15	0.2588	0.9659	0.2679	3.8637	1.0353	3.7321
16	0.2756	0.9613	0.2867	3.6280	1.0403	3.4874
17	0.2924	0.9563	0.3057	3.4208	1.0457	3.2709
18	0.3090	0.9511	0.3249	3.2361	1.0515	3.0777
19	0.3256	0.9455	0.3443	3.0716	1.0576	2.9042
20	0.3420	0.9397	0.3640	2.9238	1.0642	2.7475
21	0.3584	0.9336	0.3839	2.7904	1.0711	2.6051
22	0.3746	0.9272	0.4040	2.6695	1.0785	2.4751
23	0.3907	0.9205	0.4245	2.5593	1.0864	2.3559
24	0.4067	0.9135	0.4452	2.4586	1.0946	2.2460
25	0.4226	0.9063	0.4663	2.3662	1.1034	2.1445
26	0.4384	0.8988	0.4877	2.2812	1.1126	2.0503
27	0.4540	0.8910	0.5095	2.2027	1.1223	1.9626
28	0.4695	0.8829	0.5317	2.1301	1.1326	1.8807
29	0.4848	0.8746	0.5543	2.0627	1.1434	1.8040
30	0.5000	0.8660	0.5774	2.0000	1.1547	1.7321
31	0.5150	0.8572	0.6009	1.9416	1.1667	1.6643
32	0.5299	0.8480	0.6249	1.8871	1.1792	1.6003
33	0.5446	0.8387	0.6494	1.8361	1.1924	1.5399
34	0.5592	0.8290	0.6745	1.7883	1.2062	1.4826
35	0.5736	0.8192	0.7002	1.7435	1.2208	1.4281
36	0.5878	0.8090	0.7265	1.7013	1.2361	1.3764
37	0.6018	0.7986	0.7536	1.6616	1.2521	1.3270
38	0.6157	0.7880	0.7813	1.6243	1.2690	1.2799
39	0.6293	0.7771	0.8098	1.5890	1.2868	1.2349
40	0.6428	0.7660	0.8391	1.5557	1.3054	1.1918
41	0.6561	0.7547	0.8693	1.5243	1.3250	1.1504
42	0.6691	0.7431	0.9004	1.4945	1.3456	1.1106
43	0.6820	0.7314	0.9325	1.4663	1.3673	1.0724
44	0.6947	0.7193	0.9657	1.4396	1.3902	1.0355
45	0.7071	0.7071	1.0000	1.4142	1.4142	1.0000

ANGLE deg	sin	cos	tan	csc	sec	cot
45	0.7071	0.7071	1.0000	1.4142	1.4142	1.0000
46	0.7193	0.6947	1.0355	1.3901	1.4396	0.9657
47	0.7314	0.6820	1.0724	1.3673	1.4663	0.9325
48	0.7431	0.6691	1.1106	1.3456	1.4945	0.9004
49	0.7547	0.6561	1.1504	1.3250	1.5243	0.8693
50	0.7660	0.6428	1.1918	1.3054	1.5557	0.8391
51	0.7771	0.6293	1.2349	1.2868	1.5890	0.8098
52	0.7880	0.6157	1.2799	1.2690	1.6243	0.7813
53	0.7986	0.6018	1.3270	1.2521	1.6616	0.7536
54	0.8090	0.5878	1.3764	1.2361	1.7013	0.7265
55	0.8192	0.5736	1.4281	1.2208	1.7435	0.7002
56	0.8290	0.5592	1.4826	1.2062	1.7883	0.6745
57	0.8387	0.5446	1.5399	1.1924	1.8361	0.6494
58	0.8480	0.5299	1.6003	1.1792	1.8871	0.6249
59	0.8572	0.5150	1.6643	1.1666	1.9416	0.6009
60	0.8660	0.5000	1.7321	1.1547	2.0000	0.5774
61	0.8746	0.4848	1.8040	1.1434	2.0627	0.5543
62	0.8829	0.4695	1.8807	1.1326	2.1301	0.5317
63	0.8910	0.4540	1.9626	1.1223	2.2027	0.5095
64	0.8988	0.4384	2.0503	1.1126	2.2812	0.4877
65	0.9063	0.4226	2.1445	1.1034	2.3662	0.4663
66	0.9135	0.4067	2.2460	1.0946	2.4586	0.4452
67	0.9205	0.3907	2.3559	1.0864	2.5593	0.4245
68	0.9272	0.3746	2.4751	1.0785	2.6695	0.4040
69	0.9336	0.3584	2.6051	1.0712	2.7904	0.3839
70	0.9397	0.3420	2.7475	1.0642	2.9238	0.3640
71	0.9455	0.3256	2.9042	1.0576	3.0716	0.3443
72	0.9511	0.3090	3.0777	1.0515	3.2361	0.3249
73	0.9563	0.2924	3.2709	1.0457	3.4203	0.3057
74	0.9613	0.2756	3.4874	1.0403	3.6280	0.2867
75	0.9659	0.2588	3.7321	1.0353	3.8637	0.2679
76	0.9703	0.2419	4.0108	1.0306	4.1336	0.2493
77	0.9744	0.2250	4.3315	1.0263	4.4454	0.2309
78	0.9781	0.2079	4.7046	1.0223	4.8097	0.2126
79	0.9816	0.1908	5.1446	1.0187	5.2408	0.1944
80	0.9848	0.1736	5.6713	1.0154	5.7588	0.1763
81	0.9877	0.1564	6.3138	1.0125	6.3925	0.1584
82	0.9903	0.1392	7.1154	1.0098	7.1853	0.1405
83	0.9925	0.1219	8.1443	1.0075	8.2055	0.1228
84	0.9945	0.1045	9.5144	1.0055	9.5668	0.1051
85	0.9962	0.0872	11.4301	1.0038	11.474	0.0875
86	0.9976	0.0698	14.3007	1.0024	14.336	0.0699
87	0.9986	0.0523	19.0811	1.0014	19.107	0.0524
88	0.9994	0.0349	28.6363	1.0006	28.654	0.0349
89	0.9998	0.0175	57.2900	1.0002	57.299	0.0175
90	1.0000	0.0000	undefined	1.0000	undefined	0.0000

MATHEMATICS 23

FINAL TEST

GENERAL INSTRUCTIONS

1. The time limit for this test is **2 hours.**

2. There are **150 marks** on this exam.

   PART A: Multiple Choice - 30 marks

   PART B: Written Response - 120 marks

3. Calculators that are not programmed may be used.

4. Trigonometric tables are provided at the end of the test.

5. It is better to complete all the work you know well first. After that you can deal with the difficult ones. Do not spend too much time on any one problem. If you finish the test before the two-hour time limit, go back and check your answers.

PART A: MULTIPLE CHOICE

All multiple-choice questions must be answered on the Part A Response Page included in your test.

Read carefully and decide which of the choices BEST completes the statement or answers the question. Locate the question number on the Response Page and place the appropriate letter A, B, C, D, or E in the blank.

Example:

83. The value of -5^2 is

A. 10

B. -10

C. 25

D. -25

E. none of the above

Response Page

D 83.

1. The value of $\dfrac{(-27)^{\frac{4}{3}}}{(64)^{-\frac{1}{3}}}$ is

A. $\dfrac{16}{81}$

B. $\dfrac{81}{4}$

C. 81

D. 324

E. none of the above

2. The G.C.F. of $15x^3y^2z$, $30x^2y^2z^3$, and $60x^4y^3z^2$ is

   A. $60x^4y^3z^3$

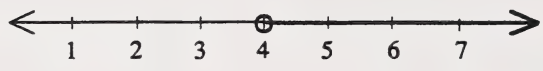
   B. $15x^2y^2z$

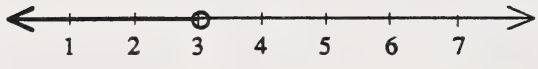
   C. $30x^4y^3z^3$

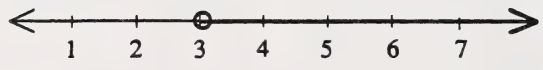
   D. $60x^2y^2z$

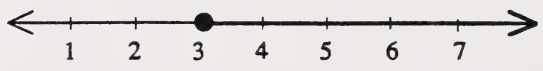
   E. none of the above

3. The graph of the solution to the inequality $-5x+7<-8$ is

   A. 

   B. 

   C. 

   D. 

   E. none of the above

4. The straight-line graph of $y = -4$ has the characteristic that it

   A. is parallel to the y-axis

   B. has an x-intercept

   C. has no y-intercept

   D. has a slope of 0

   E. none of the above

5. The slope of a line perpendicular to the graph of $y = 7x + 3$ is

   A. 7

   B. -7

   C. $\frac{1}{7}$

   D. $-\frac{1}{7}$

   E. none of the above

6. A linear equation system in which the slopes of the two equations are the same and the y-intercepts are the same has

   A. no solution

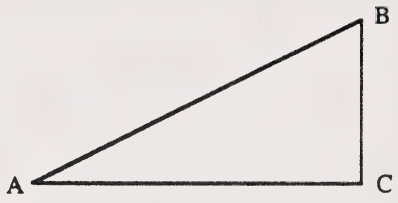
   B. one solution

   C. two solutions

   D. an infinite number of solutions

   E. none of the above

7. With respect to $\angle A$, the trigonometric ratio $\frac{BC}{AC}$ is called the



A. tangent of $\angle A$

B. cosine of $\angle A$

C. sine of $\angle A$

D. secant of $\angle A$

E. none of the above

8. The formula $A = 4\pi r^2$ finds the surface area of a

A. cone

B. cylinder

C. dome

D. sphere

E. none of the above

9. If $\sin\theta = \frac{\sqrt{3}}{2}$, what is $\cos\theta$?

A. 2

B. $\frac{1}{2}$

C. $\sqrt{3}$

D. $\frac{2}{\sqrt{3}}$

E. none of the above

10. The probability of rolling a 2, 3, or 4 using a die is

A. $\frac{3}{5}$

B. $\frac{2}{3}$

C. $\frac{1}{2}$

D. $\frac{3}{4}$

E. none of the above

11. The value of $4^{-2}\left(\frac{1}{5}\right)^{-2}(-2)^{-1}$ is

A. $\frac{25}{32}$

B. $-\frac{25}{32}$

C. $\frac{32}{25}$

D. $-\frac{32}{25}$

E. none of the above

12. The amount paid on a $9200 loan at the end of 5 years at 8% per annum compounded semiannually is approximately

A. $12 967.16

B. $13 053.32

C. $13 412.76

D. $13 618.25

E. none of the above

13. Suppose you toss a coin and record whether it lands heads (H) or tails (T). If you toss the coin 3 times, the probability of recording HTH is

A. $\frac{1}{9}$

B. $\frac{1}{8}$

C. $\frac{1}{6}$

D. $\frac{1}{3}$

E. none of the above

14. Determine which statement is NOT true.

A. The area of a trapezoid is $\frac{1}{2}h(a+b)$.

B. The volume of a sphere with a radius of 3 cm is $113\ \text{cm}^3$.

C. A pyramid is a solid that has a polygon as a base.

D. Euler's formula is $V + E - F = 2$.

E. none of the above

15. The solution to the linear system $5x - 2y = -19$ and $3x + 3y = -3$ is

A. $(3, -2)$

B. $(3, 2)$

C. $(-3, 2)$

D. $(-3, -2)$

E. none of the above

PART A: RESPONSE PAGE

_____	1.	_____	9.
_____	2.	_____	10.
_____	3.	_____	11.
_____	4.	_____	12.
_____	5.	_____	13.
_____	6.	_____	14.
_____	7.	_____	15.
_____	8.		

Name of Student _____ Student I.D. # _____

Name of School _____ Date _____

Value

PART B: WRITTEN RESPONSE

Be sure to show all your work. Part marks will be given for partially correct solutions.

1. Simplify each of the following.

(2) a. $\left(10^4\right)^2 \times 6^0$

(2) b. $\dfrac{10}{6^{-2}}$

(3) c. $\sqrt[3]{\dfrac{250}{16}}$

(2) d. $343^{\frac{2}{3}}$

(2) e. $\left(\dfrac{x^2y^2}{d^4} \times \dfrac{d^3xy^3}{y^4}\right)^2$

Name of Student _____ Student I.D. # _____

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Value

2. Factor fully.

(1) a. $8x^2y^3-24x^2y^5$

(2) b. $2x(x-6y)-7(x-6y)$

(2) c. $(x+y)^2-16$

(2) d. $8x^2+10x-3$

(2) 3. Solve $8-3x \leq -7$.

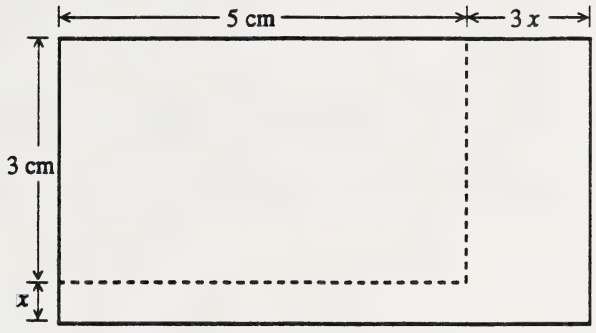
Name of Student _____ Student I.D. # _____

Name of School _____ Date _____

Value

(5)

4. A rectangular piece of metal is increased by x cm in one dimension and $3x$ cm in the other dimension. Determine the new dimensions of the metal if the new plate has an area of $84\ cm^2$.



Name of Student _____ Student I.D. # _____

Name of School _____ Date _____

Value

(4) 5. How much money must be invested today at 9% per annum compounded semiannually in order to produce $6800 in 7 years?

(2) 6. Determine the slope of the line through A(– 6, 5) and B(– 2, 9).

(3) 7. Write the equation $7x - 3y - 6 = 0$ in the form $y = mx + b$ and state the slope and the y-intercept.

Name of Student _____ Student I.D. # _____

Name of School _____ Date _____

Value

(4) 8. Determine the equation of the line that is parallel to $x = -3$ and passes through the point $(4, -8)$.

(5) 9. Write the equation of the line that is perpendicular to $y = -5x + 7$ and passes through the point $(2, -8)$. Write the equation in the general form $ax + by + c = 0$.

Name of Student _____ Student I.D. # _____

Name of School _____ Date _____

Value

(3) 10. The sum of two numbers is 32. Two times the smaller number is 10 more than the larger number. Determine the two numbers.

(5) 11. A rectangular field has a perimeter of 4080 m. The length is 240 m less than twice the width. Determine the length and width of the field. Use a final statement in your answer.

Name of Student _____ Student I.D. # _____

Name of School _____ Date _____

Value

(6) 12. Solve the system.

$$6x - 4y = 3$$
$$9x + 8y = 1$$

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Name of School _____ Date _____

Value

(6) 13. Four beefburgers and three glasses of milk cost $8.50. Two beefburgers and one glass of milk cost $3.90. Determine the cost of one beefburger and the cost of one glass of milk.

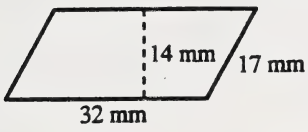
Name of Student _____ Student I.D. # _____

Name of School _____ Date _____

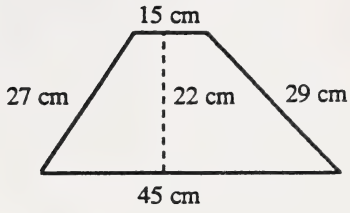
Value

(10) 14. Calculate the following areas.

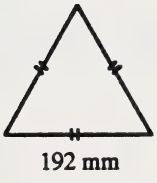
a.



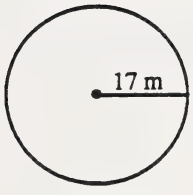
b.



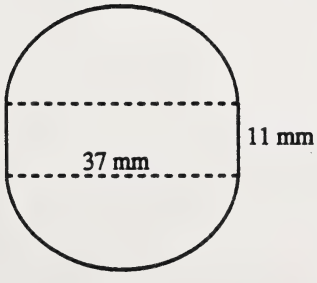
c.



d.



e.

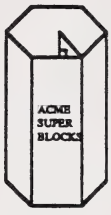


Name of Student _____ Student I.D. # _____

Name of School _____ Date _____

Value

(5) 15. What volume will be displaced by a regular-hexagonal prism that is used for packaging toy building blocks? Each side of the hexagon is 8 cm. The radius of the hexagon is 7 cm. The height of the container is 22 cm.



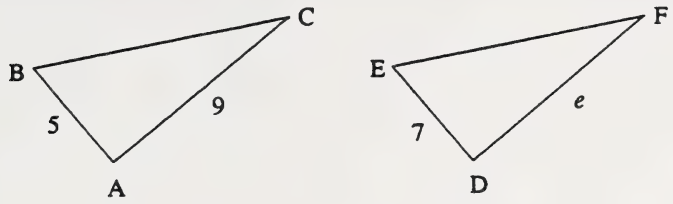
(5) 16. What is the diameter of the moon Europa if it has a surface area of 30 270 000 km^2?

Name of Student _____ Student I.D. # _____

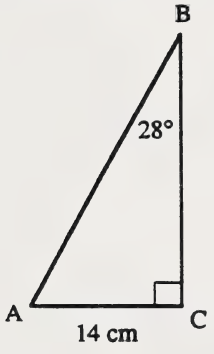
Name of School _____ Date _____

Value

(4) 17. Given $\Delta ABC \sim \Delta DEF$, determine the value of e to one decimal place.



(5) 18. Given a right triangle ABC, where $\angle B = 28°$ and side $b = 14$ cm, determine the length of the hypotenuse of the triangle. Round the final answer to two decimal places.



Name of Student _____ Student I.D. # _____

Name of School _____ Date _____

Value

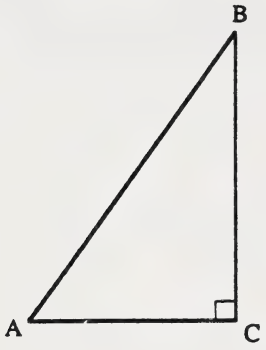
(5) 19. A post 1.8 m tall casts a shadow 2.9 m along the ground. At the same time, a building has a shadow 35.8 m long. Determine the height of the building to one decimal place.

Name of Student _____ Student I.D. # _____

Name of School _____ Date _____

Value

(6) 20. Solve ΔABC if $\angle C = 90°$, BA = 8.7 cm, and BC = 6.3 cm. Round the angle measurements to a whole number and the side measurement to one decimal place.



Name of Student _____ Student I.D. # _____

Name of School _____ Date _____

Value

(5) 21. There are 15 cards of equal size. Each card has a number from 1 to 15. The cards are placed in a hat and mixed. Determine the theoretical probabilities for the following situations.

a. What is the probability of drawing a number greater than 11?

b. What is the probability of drawing an odd number?

c. What is the probability of drawing a prime number?

d. What is the probability of drawing a number greater than 6 but less than 12?

e. What is the probability of drawing a number less than 6 and greater than 10?

Name of Student _____ Student I.D. # _____

Name of School _____ Date _____

Value

22. Use the information in the table to answer the following questions.

Math Marks	
Mark (%)	Frequency
0 - 40	4
41 - 45	6
46 - 50	5
51 - 55	7
56 - 60	8
61 - 65	12
66 - 70	13
71 - 75	12
76 - 80	9
81 - 85	8
86 - 90	2
91 - 95	2
96 - 100	1
Total	89

(1) a. What is the probability that a mark was above 90%?

(2) b. What is the probability that a mark was from 51 to 75?

(2) c. A mark over 50 but less than 66 is a C standing. What percent of the students receive C's?

(2) d. If 118 students wrote the exam, determine how many students would receive a mark from 66 to 80%.

Name of Student _____ Student I.D. # _____

Name of School _____ Date _____

Value

23. An office worker has ten different hats. This worker decides to randomly select a hat to wear to the office each day for the five-day work week. Use a simulation to answer the following questions. Use the random number table of 50 trials given below and let the digits 0 to 9 each represent a different hat. Each block of 5 will represent a five-day week. To answer the questions, each trial must be checked.

52097	96239	72255	86588	13395	60424	63156	92309	79545	64637
58233	53891	61336	38086	39495	37774	27949	34573	58358	54006
10884	42540	17696	86020	78016	38734	04767	49518	19256	24990
35593	83183	91244	68292	24406	06991	47542	18392	25866	76750
67050	00008	52103	59222	26126	74210	01129	15483	86292	53468

(3) a. Estimate the probability that this worker will wear a different hat each day.

(1) b. Estimate the probability that this worker will wear the same hat each day.

(1) c. Estimate the probability that this worker will wear the same hat more than once in one week.

Name of Student _____ Student I.D. # _____

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TABLE OF TRIGONOMETRIC FUNCTIONS

ANGLE deg	sin	cos	tan	csc	sec	cot
0	0.0000	1.0000	0.0000	undefined	1.0000	undefined
1	0.0175	0.9998	0.0175	57.299	1.0002	57.2900
2	0.0349	0.9994	0.0349	28.654	1.0006	28.6363
3	0.0523	0.9986	0.0524	19.107	1.0014	19.0811
4	0.0698	0.9976	0.0699	14.336	1.0024	14.3007
5	0.0872	0.9962	0.0875	11.474	1.0038	11.4301
6	0.1045	0.9945	0.1051	9.5668	1.0055	9.5144
7	0.1219	0.9925	0.1228	8.2055	1.0075	8.1443
8	0.1392	0.9903	0.1405	7.1853	1.0098	7.1154
9	0.1564	0.9877	0.1584	6.3925	1.0125	6.3138
10	0.1736	0.9848	0.1763	5.7588	1.0154	5.6713
11	0.1908	0.9816	0.1944	5.2408	1.0187	5.1446
12	0.2079	0.9781	0.2126	4.8097	1.0223	4.7046
13	0.2250	0.9744	0.2309	4.4454	1.0263	4.3315
14	0.2419	0.9703	0.2493	4.1336	1.0306	4.0108
15	0.2588	0.9659	0.2679	3.8637	1.0353	3.7321
16	0.2756	0.9613	0.2867	3.6280	1.0403	3.4874
17	0.2924	0.9563	0.3057	3.4208	1.0457	3.2709
18	0.3090	0.9511	0.3249	3.2361	1.0515	3.0777
19	0.3256	0.9455	0.3443	3.0716	1.0576	2.9042
20	0.3420	0.9397	0.3640	2.9238	1.0642	2.7475
21	0.3584	0.9336	0.3839	2.7904	1.0711	2.6051
22	0.3746	0.9272	0.4040	2.6695	1.0785	2.4751
23	0.3907	0.9205	0.4245	2.5593	1.0864	2.3559
24	0.4067	0.9135	0.4452	2.4586	1.0946	2.2460
25	0.4226	0.9063	0.4663	2.3662	1.1034	2.1445
26	0.4384	0.8988	0.4877	2.2812	1.1126	2.0503
27	0.4540	0.8910	0.5095	2.2027	1.1223	1.9626
28	0.4695	0.8829	0.5317	2.1301	1.1326	1.8807
29	0.4848	0.8746	0.5543	2.0627	1.1434	1.8040
30	0.5000	0.8660	0.5774	2.0000	1.1547	1.7321
31	0.5150	0.8572	0.6009	1.9416	1.1667	1.6643
32	0.5299	0.8480	0.6249	1.8871	1.1792	1.6003
33	0.5446	0.8387	0.6494	1.8361	1.1924	1.5399
34	0.5592	0.8290	0.6745	1.7883	1.2062	1.4826
35	0.5736	0.8192	0.7002	1.7435	1.2208	1.4281
36	0.5878	0.8090	0.7265	1.7013	1.2361	1.3764
37	0.6018	0.7986	0.7536	1.6616	1.2521	1.3270
38	0.6157	0.7880	0.7813	1.6243	1.2690	1.2799
39	0.6293	0.7771	0.8098	1.5890	1.2868	1.2349
40	0.6428	0.7660	0.8391	1.5557	1.3054	1.1918
41	0.6561	0.7547	0.8693	1.5243	1.3250	1.1504
42	0.6691	0.7431	0.9004	1.4945	1.3456	1.1106
43	0.6820	0.7314	0.9325	1.4663	1.3673	1.0724
44	0.6947	0.7193	0.9657	1.4396	1.3902	1.0355
45	0.7071	0.7071	1.0000	1.4142	1.4142	1.0000

ANGLE deg	sin	cos	tan	csc	sec	cot
45	0.7071	0.7071	1.0000	1.4142	1.4142	1.0000
46	0.7193	0.6947	1.0355	1.3901	1.4396	0.9657
47	0.7314	0.6820	1.0724	1.3673	1.4663	0.9325
48	0.7431	0.6691	1.1106	1.3456	1.4945	0.9004
49	0.7547	0.6561	1.1504	1.3250	1.5243	0.8693
50	0.7660	0.6428	1.1918	1.3054	1.5557	0.8391
51	0.7771	0.6293	1.2349	1.2868	1.5890	0.8098
52	0.7880	0.6157	1.2799	1.2690	1.6243	0.7813
53	0.7986	0.6018	1.3270	1.2521	1.6616	0.7536
54	0.8090	0.5878	1.3764	1.2361	1.7013	0.7265
55	0.8192	0.5736	1.4281	1.2208	1.7435	0.7002
56	0.8290	0.5592	1.4826	1.2062	1.7883	0.6745
57	0.8387	0.5446	1.5399	1.1924	1.8361	0.6494
58	0.8480	0.5299	1.6003	1.1792	1.8871	0.6249
59	0.8572	0.5150	1.6643	1.1666	1.9416	0.6009
60	0.8660	0.5000	1.7321	1.1547	2.0000	0.5774
61	0.8746	0.4848	1.8040	1.1434	2.0627	0.5543
62	0.8829	0.4695	1.8807	1.1326	2.1301	0.5317
63	0.8910	0.4540	1.9626	1.1223	2.2027	0.5095
64	0.8988	0.4384	2.0503	1.1126	2.2812	0.4877
65	0.9063	0.4226	2.1445	1.1034	2.3662	0.4663
66	0.9135	0.4067	2.2460	1.0946	2.4586	0.4452
67	0.9205	0.3907	2.3559	1.0864	2.5593	0.4245
68	0.9272	0.3746	2.4751	1.0785	2.6695	0.4040
69	0.9336	0.3584	2.6051	1.0712	2.7904	0.3839
70	0.9397	0.3420	2.7475	1.0642	2.9238	0.3640
71	0.9455	0.3256	2.9042	1.0576	3.0716	0.3443
72	0.9511	0.3090	3.0777	1.0515	3.2361	0.3249
73	0.9563	0.2924	3.2709	1.0457	3.4203	0.3057
74	0.9613	0.2756	3.4874	1.0403	3.6280	0.2867
75	0.9659	0.2588	3.7321	1.0353	3.8637	0.2679
76	0.9703	0.2419	4.0108	1.0306	4.1336	0.2493
77	0.9744	0.2250	4.3315	1.0263	4.4454	0.2309
78	0.9781	0.2079	4.7046	1.0223	4.8097	0.2126
79	0.9816	0.1908	5.1446	1.0187	5.2408	0.1944
80	0.9848	0.1736	5.6713	1.0154	5.7588	0.1763
81	0.9877	0.1564	6.3138	1.0125	6.3925	0.1584
82	0.9903	0.1392	7.1154	1.0098	7.1853	0.1405
83	0.9925	0.1219	8.1443	1.0075	8.2055	0.1228
84	0.9945	0.1045	9.5144	1.0055	9.5668	0.1051
85	0.9962	0.0872	11.4301	1.0038	11.474	0.0875
86	0.9976	0.0698	14.3007	1.0024	14.336	0.0699
87	0.9986	0.0523	19.0811	1.0014	19.107	0.0524
88	0.9994	0.0349	28.6363	1.0006	28.654	0.0349
89	0.9998	0.0175	57.2900	1.0002	57.299	0.0175
90	1.0000	0.0000	undefined	1.0000	undefined	0.0000

Teacher Questionnaire

Course name & number _____ Teacher's name _____

Teacher's area of expertise _____ School name _____

Date _____

Design

1. The modules follows a definite systematic design. Did you find it easy to follow?

   Yes No If no, explain.

   _____

   _____

2. Did your observations reveal that the students found the design easy to follow?

   Yes No If no, explain.

   _____

   _____

3. Did you find the Learning Facilitator's Manual helpful?

   Yes No If no, explain.

   _____

   _____

4. Part of the design involves stating the objectives in student terms. Do you feel this helped the students understand what they were going to learn?

   Yes No If no, explain.

   _____

   _____

5. The activities include self-assessment questions. These questions are to help clarify and reinforce the instructional materials. Did students use the answers effectively?

Yes No If no, explain.

_____

_____

6. Did the two streams of Follow-up Activites (Extra-help and Enrichment) prove to be helpful?

Yes No If no, explain.

_____

_____

7. Were students motivated to try these Follow-up Activities on their own?

Yes No If no, give details.

_____

_____

8. Were the assignments clear?

Yes No If no, give details.

_____

_____

9. Were the assignments appropriate?

Yes No If no, give details.

_____

_____

10. Did you fax assignments?

Yes No

11. If you did fax, did you get satisfactory results from using this procedure?

Yes No If no, give details.

_____

_____

_____

Instruction

1. Did you find the instruction clear?

Yes No If no, give details.

_____

_____

2. Did your observations reveal that the students found the instruction interesting?

Yes No If no, give details.

_____

_____

3. Did you find the instruction adequate?

Yes No If no, give details.

_____

_____

4. Was the reading level appropriate?

Yes No If no, give details.

_____

_____

5. Was the work load adequate?

Yes No If no, give details.

_____

_____

6. Was the content accurate and current?

Yes No If no, give details.

_____

_____

7. Did the content flow consistently and logically?

Yes No If no, give details.

_____

_____

8. Was the transition between booklets smooth?

Yes No If no, give details.

_____

_____

9. Was the transition between print and media smooth?

Yes No If no, give details.

Additional Comments

When you have completed this questionnaire, please mail it to the following address.

Instructional Design and Development
Alberta Distance Learning Centre
Box 4000
Barrhead, Alberta
T7N 1P4

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